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SPACE TUG ECONOMIC ANALYSIS STUDY

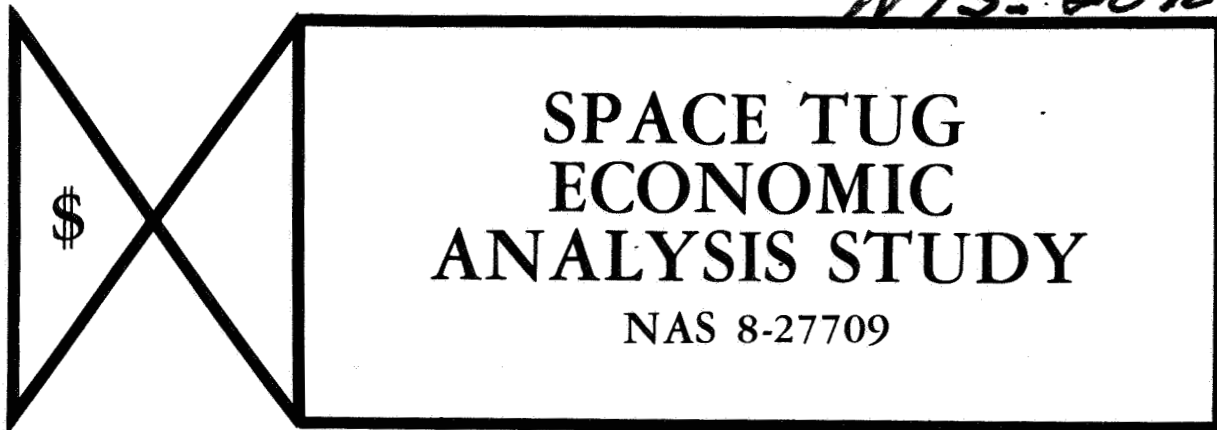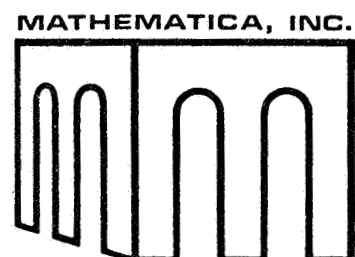## NAS 8-27709

FINAL REPORT
DR MA-04

CASE FILE COPY

VOLUME II: TUG CONCEPTS ANALYSIS

PART 2 — ECONOMIC ANALYSIS

Prepared for

National Aeronautics & Space Administration
George C. Marshall Space Flight Center

Lockheed Missiles & Space Company, Inc.
Sunnyvale, California
and
Mathematica Inc.
Princeton, New Jersey

LOCKHEED

NASA

MATHEMATICA, INC.

SPACE TUG

ECONOMIC ANALYSIS STUDY

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May 1972

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Part 2: Economic Analysis

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FOREWORD

This report summarizes work accomplished under the Space Tug Economic Analysis Study on Contract NAS8-27709. This study was performed for the NASA Marshall Space Flight Center by Lockheed Missiles & Space Company, Inc. of Sunnyvale, California, and Mathematica, Inc. of Princeton, New Jersey. The period of technical performance was nine months, starting July 26, 1971.

The NASA Contracting Officer's Representatives for this program were Lieutenant Commander William C. Stilwell (USN) and Mr. Richard L. Klan. The study team was led by Mr. Charles V. Hopkins of Lockheed and Dr. Edward Greenblat of Mathematica.

Key Mathematica team members included:

>   Dr. Leonard Jacobson   —   Operations Analysis
>   Mrs. Donna Mazzola   —   Computer Programming

This report is organized as follows:

- Volume I   — Executive Summary
- Volume II   — Tug Concepts Analysis
    Part 1: Overall Approach and Data Generation
    Part 2: Economic Analysis
    Appendix: Tug Design and Performance Data Base
- Volume III   — Cost Estimates

Volume II contains detailed discussions of the methods used to perform this study, and of the major findings that have resulted. For convenience Volume II has been further divided into three parts. Part 2 is devoted exclusively to details of the economic analysis performed by Mathematica.

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Chapter 5

INTRODUCTION TO THE ECONOMIC ANALYSIS

Part 2 of Volume II reports in detail the approach and results of the economic analysis performed by Mathematica Inc. Part 2 contains three chapters; these are numbered consecutively following the four chapters of Part 1. Two annexes appended to Part 2 discuss background and peripheral issues of the economic analysis.

That the Space Tug would play a vital economic role in the cost effectiveness of the Space Shuttle/Tug transportation system was anticipated at the beginning of this study. This understanding was derived from earlier economic studies of the proposed new space transportation system (STS). These studies indicated that a vehicle that provided for recoverability and reuse of payloads would be an economically cost effective project for the United States Government. The results of the Tug Economic Analysis study have confirmed this. A thorough investigation of the impact of a reusable Tug on the total program costs of payloads, through the reuse capability and the so-called mass and volume effects, reveals that a Space Tug could return more than $1 billion in RDT&E funds (above its development costs) to the economic justification of a Space Shuttle.

The Mathematica task in the Tug Economic Analysis study was to provide the methodology and analysis upon which a Tug or family of Tugs could be selected economically from the candidates provided by LMSC. The emphasis of the analysis was upon a parametric approach to determine the impact on Tug choice of variations in cost and in the mission model. The major tasks accomplished by Mathematica in the parametric approach to the economic analysis are indicated in Table 5-1. Using raw data supplied by LMSC (in punched card format) and the Mathematica computer program, SCENARIO, the economic analysis included attention to sensitivity analyses of the variables indicated in Table 5-1. As indicated in the table, the sensitivity analysis included nonrecurring costs (RDT&E and, in the case of the reusable Tugs, fleet acquisition costs), and operations costs (including those costs that are said to be activity level dependent, and those that are independent of the level of activity). RDT&E and unit costs were included

5-1

Table 5-1 SENSITIVITY ANALYSES CONDUCTED BY MATHEMATICA

```
TUG RDT&E COST
TUG UNIT COST
TUG OPERATIONS COST
    ● ACTIVITY LEVEL DEPENDENT
    ● ACTIVITY LEVEL INDEPENDENT
PAYLOAD RDT&E COST
PAYLOAD UNIT COST
PAYLOAD REFURBISHMENT COST
PAYLOAD OPERATIONS COST
    ● ACTIVITY LEVEL DEPENDENT
    ● ACTIVITY LEVEL INDEPENDENT
SHUTTLE ACTIVITY-LEVEL-DEPENDENT COST
ACTIVITY VARIABLES
    ● AGENCY
    ● ENERGY PROFILE
    ● LIFETIME
```

and most importantly, payload refurbishment costs. A major factor in the cost effectiveness of the Space Shuttle/Tug system is the capability to recover and to refurbish, for reuse, payloads in various orbits. The cost of refurbishing payloads, therefore, is a key variable, and a sensitivity analysis of this parameter should cover a wide range of values. The other important sensitivity analyses include those pertaining to the Space Shuttle user fee, and the mission model; the latter is broken down into elements called activity variables.

The activity variables represent a major uncertainty for the economic analysis. The mission model, as provided for this study, projects NASA, DoD, and other non-NASA space activities into the 1980s. It is at best a planning document of what may be done in the timeframe of the 1980s. The major source of uncertainty in the mission model is the inability to predict with confidence, missions that will occur in 1980, much less 1990. To be sure, some missions or activities are probably more certain than others.

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The DoD, for example, can project certain classes of missions into the indefinite future. Likewise, the OSS planetary program in part corresponds to certain predictable solar system events such as the planetary geometry and solar activity. The exact nature of the majority of missions, however, will depend upon the state of future science and technology, and the level of funding and the composition of these programs will depend upon future states of the economy, technology, and national priorities.

The computer program SCENARIO provides for parametric variations in the mission model with respect to using agency (NASA, DoD, and other non-NASA applications), the energy profile (a partition of the mission model into ideal velocities up to 3000 feet per second, from 3000 feet per second up to synchronous equatorial, and velocities greater than synchronous equatorial) and satellite lifetime (less than 1 year, 1 and 2 years, 3 and 4 years, and 5 years and more). These parametric variations of the activity variables may be used either alone or in combination, e.g., we may wish to determine the impact on the cost effectiveness of the alternative Space Tug and orbital injection stages by examining what happens if we double the amount of non-NASA applications missions at synchronous equatorial orbits. By systematically exploring these activity variables, we are able to include a broad range of possible unmanned future space activities and thereby remove the constraint of having our analysis tied to a given mission model.

The Space Tug Economic Analysis study was divided into two phases. The object of Phase I was to determine the relative cost effectiveness of a large number of orbital injection stages and reusable Space Tug candidates, the most promising of which were to be carried into Phase II for further refinement and study. Part 2 of Volume II is organized accordingly, with a review of the work performed by Mathematica in each Phase. There are also two annexes, one giving an overview of the economic methodology applied to this study and the other giving a detailed description of a mixed integer programming approach to Tug selection taken as an expansion of the economic analysis.

Chapter 6
PHASE I ECONOMIC ANALYSIS

For Phase I of the analysis, Mathematica conducted sensitivity analyses on the Space Tug and orbit injection stage (OIS) concepts presented in Table 6-1. These included the reference case, which was the Agena/Centaur OIS family with no payload mass and volume savings assumed; a number of LO_2/LH_2 reusable Tug candidates with propellant loadings from 36,000 lb to 58,000 lb; LF_2/LH_2 and $FLOX/CH_4$ reusable Tugs of a range of propellant loadings; and reusable Tugs boosted by lower and higher capability Space Shuttles.

A first cut at the Phase I economic analysis is shown in Figure 6-1 where the candidate orbit injection stages and Space Tugs are shown in an economic relationship. The abscissa of this graph is the present value (discounted at 10 percent) of foregone recurring-cost savings, and the ordinate is the present value of total nonrecurring costs (RDT&E and Tug fleet investment). The present value of foregone recurring-costs savings for a given candidate is defined as the difference between the recurring costs of the given OIS or Space Tug and the recurring costs of that Tug requiring the least recurring costs to perform the mission model. In Figure 6-1 candidates 8, 9, and 10 require the least cost to perform the mission model. The difference between the recurring cost of these candidates and, for example, the reference case (the Agena/ Centaur family without payload effects) is approximately $2.2 billion in 1970 dollars discounted at 10 percent. The Agena/Centaur orbital injection stage family, therefore, foregoes $2.2 billion in present values, in potential savings. This is because it does not have the capability to retrieve payloads for refurbishment and reuse, and also because payload mass and volume effects have not been considered. The ordinate, as mentioned before, represents the present value (10 percent discounted) of the nonrecurring costs, i.e., RDT&E and nonrecurring investment.

Table 6-1 TUG CASES ANALYZED BY MATHEMATICA FOR PHASE I

CASE	PAYLOAD MASS-VOLUME EFFECTS	PAYLOAD REFURBISHMENT
AGENA/CENTAUR	NO	NO
AGENA/CENTAUR	YES	NO
CENTAUR	NO	NO
CENTAUR	YES	NO
LARGE TANK AGENA (LTA)	YES	NO
36K LO_2/LH_2	YES	YES
50K LO_2/LH_2	YES	YES
58K LO_2/LH_2	YES	YES
48K LF_2/LH_2	YES	YES
54K LF_2/LH_2	YES	YES
59K $FLOX/CH_4$	YES	YES
36K LO_2/LH_2 LOWER CAPABILITY SHUTTLE	YES	YES
58K LO_2/LH_2 HIGHER CAPABILITY SHUTTLE	YES	YES

RECURRING VS NONRECURRING COSTS
(Billions of 1970 Dollars, 10%)

1 = AGENA/CENTAUR, NO PLE
2 = AGENA/CENTAUR, PLE
3 = CENTAUR, NO PLE
4 = CENTAUR, PLE
5 = 36K LO_2/LH_2, PLE
6 = 50K LO_2/LH_2, PLE
7 = 58K LO_2/LH_2, PLE
8 = 48K LF_2/LH_2, PLE
9 = 54K LF_2/LH_2, PLE
10 = 60K LF_2/LH_2, PLE
11 = 58K $FLOX/CH_4$, PLE
12 = LARGE TANK AGENA, PLE
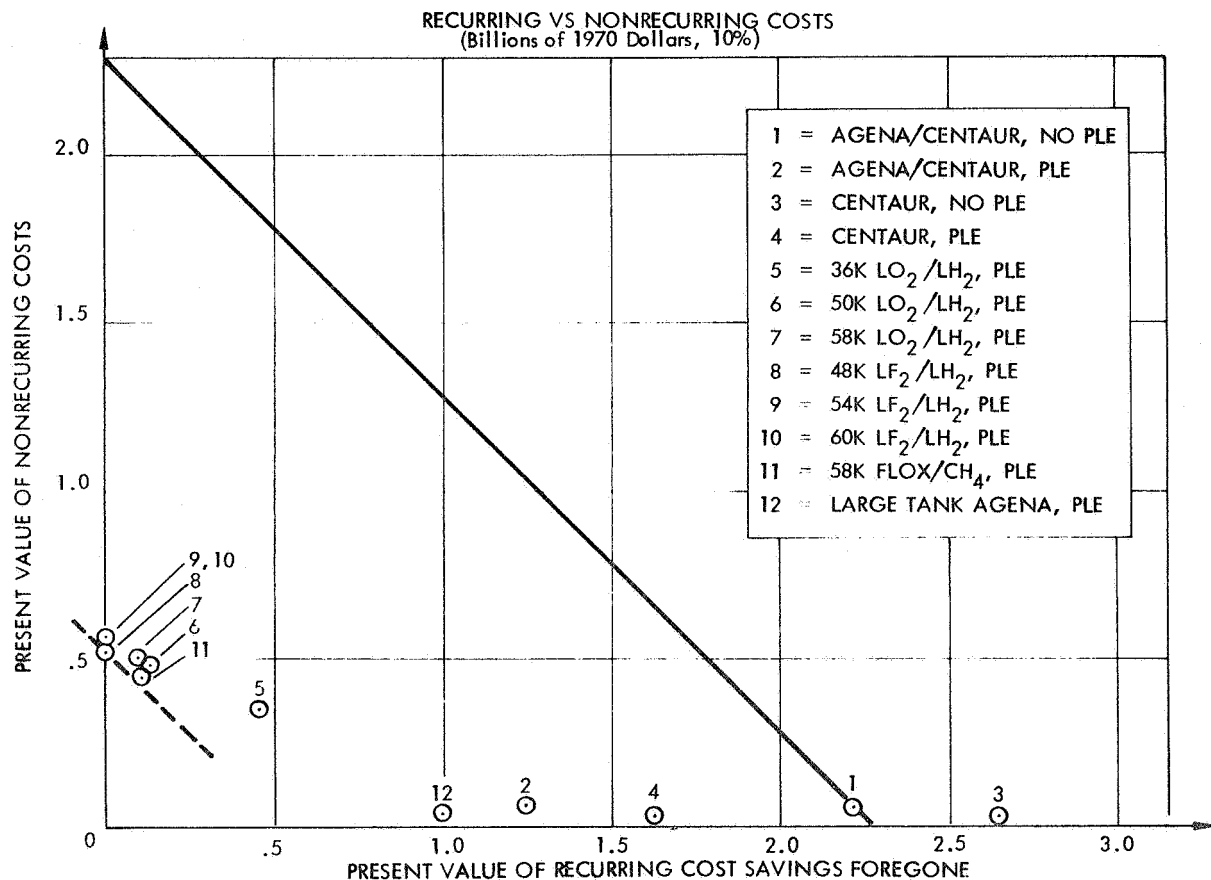
Figure 6-1 Space Tug Economic Analysis

The economic superiority of a candidate system is measured by the distance of the orthogonal projection of each candidate in Figure 6-1 to the heavy diagonal line representing the economic tradeoff function defined by Family 1, the reference case. Any candidate that maps onto the line is economically equal to the reference case. Any candidate that maps to the right of the tradeoff function such as Family 3 is economically inefficient; any family lying to the left (as most do in our analysis) is economically efficient with respect to the reference case.

As presented in Figure 6-1, candidate 8, the LF_2/LH_2 48,000 lb Tug is economically superior to candidates 9 and 10, the 54,000 lb and 60,000 lb LF_2/LH_2 Tugs respectively, even though each provides the maximum in recurring cost savings. This is because candidate 8, the 48,000 lb LF_2/LH_2 Tug requires somewhat smaller outlays for RDT&E and nonrecurring investment.

Candidate 11, the 58,000 lb $FLOX/CH_4$ Tug appears to be roughly equivalent economically to candidate 8 because, even though it does not yield the maximum recurring-cost savings, candidate 11 requires less nonrecurring costs to develop and produce. Therefore, the distance from each to the economic tradeoff function is approximately the same, and they are viewed as economically indifferent.

At the outset, therefore, it appears that the 48,000 lb LF_2/LH_2 and 58,000 lb $FLOX/CH_4$ Tugs are, from the standpoint of cost effectiveness, superior. But, of course, the results shown in Figure 6-1 represent only the nominal values of the proposed Tug and orbital injection stage candidates, and it was necessary to await the results of extensive sensitivity analyses before recommendations were made concerning which candidates in Phase I appear to be the most economical.

Cost Uncertainties

Figure 6-2 illustrates the effects of uncertainties in payload cost on the economic analysis of the OIS and Space Tug candidates. Because payloads constitute the greatest part of total program costs, the impact of payload cost uncertainties on Tug cost effectiveness requires careful analysis. It bears restatement that the cost effectiveness

of a Tug is derived largely from its capability to permit reuse of payloads. Should, for some reason, the cost of payloads decline in the future, the residual value of the payloads that are being reused will be reduced and the reuse capability provided by the Shuttle/Tug system will be of reduced economic value.

The payloads in the high energy mission model were designated by NASA to be well, fair, or poorly defined. Accordingly, Mathematica assumed uncertainty factors of 1.10, 1.20, 1.30 for the three classes, respectively. To perform a sensitivity analysis, these factors were associated with the major elements in the payload life-cycle costs. As payload costs, especially unit investment, are driven upward by the uncertainty factors, the economic benefits from payload reuse are increased. This is demonstrated in Figure 6-2 with the reference Agena/Centaur orbit injection stage family foregoing an additional \$500 million in net present values moving from point 1 on the graph to 1'. As shown this serves to move the economic tradeoff function as well. The 30,000 lb LO_2/LH_2 Tug loses less in present values, moving from 5 to 5'. This is because all of the reusable Tugs provide benefits from payload reuse capabilities, and, as payload costs rise, there is only a minor economic deterioration relative to the Tugs that provide the maximum reuse capability. With respect to payload cost uncertainties, the 48,000 lb LF_2/LH_2 Tug still provides the greatest recurring cost savings benefits. However, as indicated, the 58,000 lb LO_2/LH_2 (moving from 7 to 7' as indicated in Figure 6-2) becomes roughly equivalent from a total cost effectiveness standpoint to the 48,000 lb LF_2/LH_2 Tug; this result is unique to the parametric analysis.

A set of subjective factors was generated by LMSC to quantify the uncertainty range in the RDT&E, Investment and Operations costs generated for the candidate Space Tug and orbit injection stage concepts. Table 6-2 presents these uncertainty factors. These factors were given as plus and minus percentages to be applied to the costs calculated in the LMSC parametric cost model for Space Tugs and orbit injection stages. The factors were devised on a total-system level by formulating a judgement as to the relative state-of-the-art of the Tugs, the degree of their definition, and the sources of data available. As indicated in Table 6-2, the greatest uncertainties were identified for the LF_2/LH_2 and $FLOX/CH_4$ reusable Tugs throughout their RDT&E, Investment and Operations phases. The smallest uncertainties, of course, were identified with the

(Billions of 1970 Dollars, 10%)
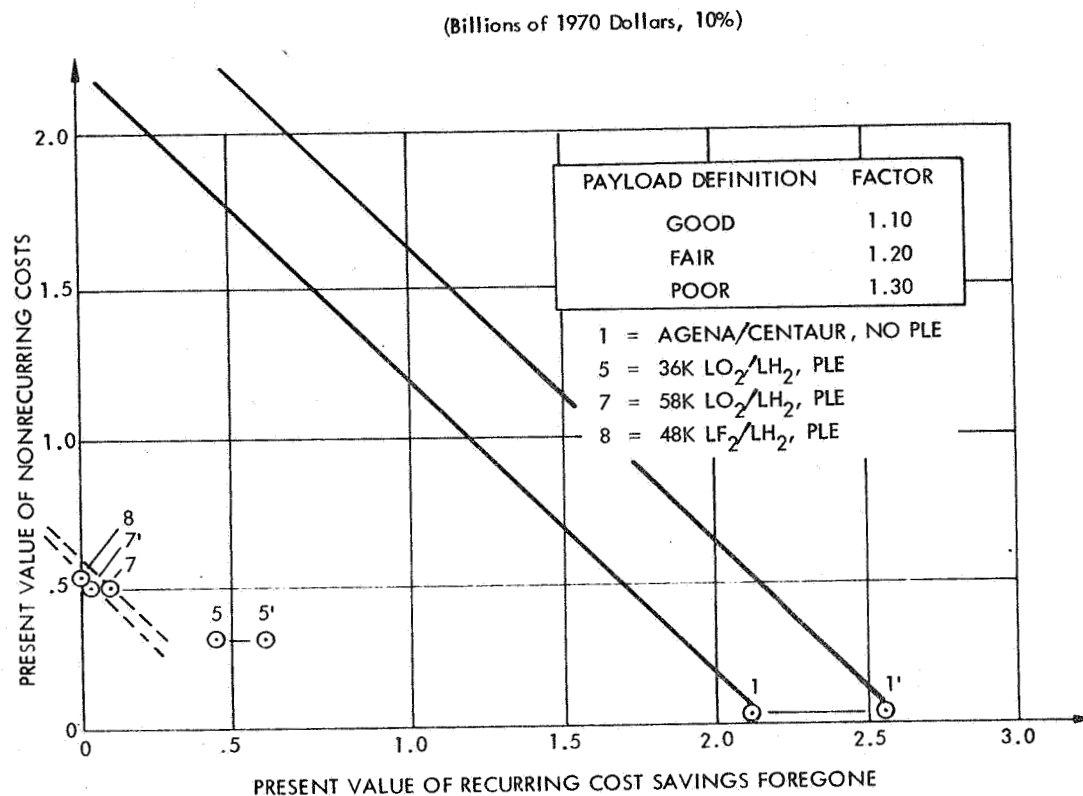
Figure 6-2 Recurring vs Nonrecurring Costs with Payload
Uncertainty Factors (Phase I)

Table 6-2 TUG COST UNCERTAINTY FACTORS USED FOR ANALYSIS

TUG CONCEPT	UNCERTAINTY (%)					
	RDT&E		INVESTMENT		ACTIVITY-LEVEL DEPENDENT OPERATIONS	
	−	↓	−	↓	−	↓
EXISTING EXPENDABLE STAGES	5	10	5	5	5	5
GROWTH VERSIONS OF EXISTING EXPENDABLES	5	15	5	5	5	5
REUSABLE LO_2/LH_2 (RL10 ENGINE)	10	35	10	15	20	100
REUSABLE LO_2/LH_2 (ALL OTHER)	5	40	10	20	20	100
REUSABLE LF_2/LH_2	0	60	10	25	20	125
REUSABLE $FLOX/CH_4$	0	60	10	25	20	125

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existing expendable orbital injection stages, with an intermediate uncertainty associated with the reusable Tugs that might use a reusable version of the existing RL10 engine. Somewhat higher uncertainties were associated with more advanced, higher performance, LO_2/LH_2 engines.

Using the uncertainty factors given in Table 6-2, Figure 6-3 shows a plot of the economic impact of the upper, nominal, and lower bounds of typical Tug costs. As shown, the impact of the uncertainty factors falls almost entirely on the nonrecurring costs. The effect of the upper limit is to make the 58,000 lb LO_2/LH_2 Tug approach the cost effectiveness of the 48,000 lb LF_2/LH_2 tug.

Sensitivity Studies

As a ground-rule of this study, it was assumed that the Space Shuttle user fee was $5 million per launch. At the time the study started, the two-stage, fully reusable Shuttle was still in the NASA plan, and therefore, a $5 million user fee was appropriate. However, over the course of this study, authorization was given for a partically expendable Shuttle, resulting in a higher user fee on the order of $10.5 million per launch. Figure 6-4 illustrates the impact of the Shuttle user fee on the cost effectiveness of the various Tug and orbit injection stage candidates. The ordinate is the present value of the total program cost savings in billions of 1970 dollars discounted at 10 percent. As mentioned above, the reference case is the Agena/Centaur OIS family without payload effects; and the present value of total program cost savings is defined as the difference between the total cost of the reference case and a given Tug candidate. The difference in the present value of total program cost between the reference case and an alternative is the net present value of a project; and any project with a net present value equal to or greater than zero at a given discount rate is said to be a cost effective project at that discount rate. These costs include the present value of all recurring expenditures extended to the infinite horizon for each of the candidates considered in Phase I of this study. The abscissa includes a range of Shuttle user fees from $5 million, the nominal value, to approximately $10.5 million per launch. As shown in Figure 6-4, variations in the Space Shuttle user fee have the strongest impact on Tugs with the greatest propellant loadings. These Tugs require the most Shuttle flights and therefore lose out economically to their smaller counterparts as the Shuttle user fee rises. Indeed, the

(Billions of 1970 Dollars, 10%)
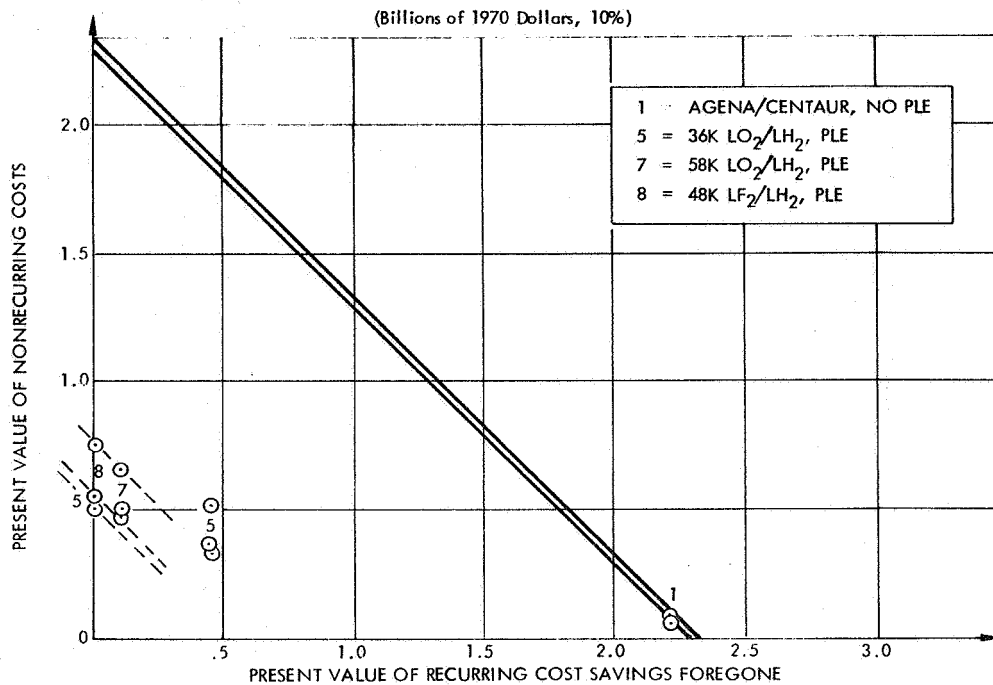
Figure 6-3 Recurring vs Nonrecurring Costs with Space
Tug Uncertainty Factors (Phase I)
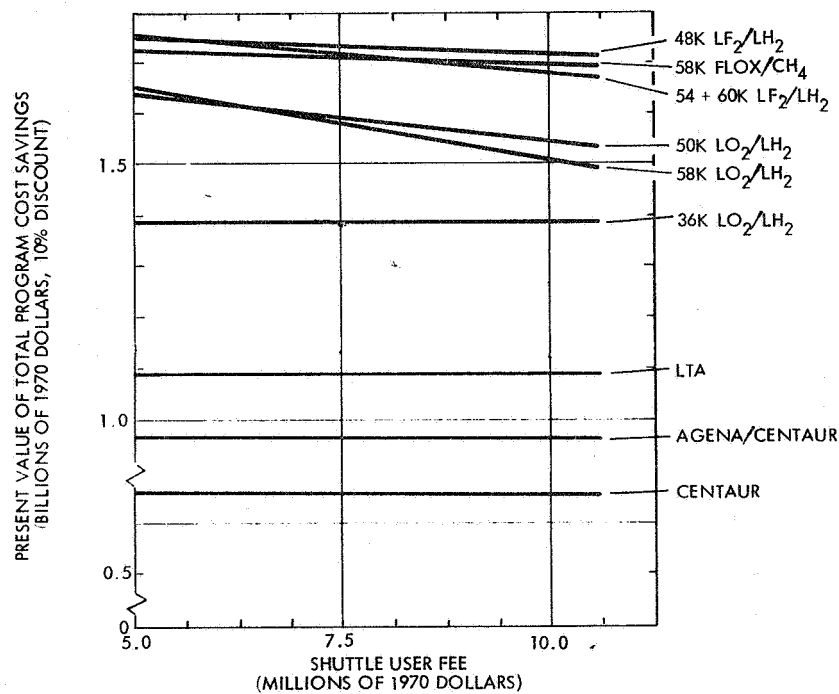
Figure 6-4 Phase I Sensitivity Analysis: Space Shuttle User Fee

cost-effectiveness crossovers occur between $6 million and $8 million per Shuttle flight which is less than the current expectation of $10.5 million per launch. The lines that apply to the Large Tank Agena, Agena/Centaur with payload effects, and Centaur with payload effects, are parallel to the abscissa because these require the same number of Shuttle flights as the reference case, (Agena/Centaur without payload effects). Therefore, as the Shuttle user fee rises, their relative economic standing is unaffected.

A sensitivity analysis to the Space Tug and orbit injection stage user fee is illustrated in Figure 6-5. Although there are other cost elements that make up the Tug user fee, the sensitivity analysis, as illustrated here, was intended primarily to reflect variations in Space Tug refurbishment costs. As illustrated in Figure 6-5, the Space Tug user fee impacts the 36,000 lb LO_2/LH_2 Tug the strongest because the 36,000 lb Tug requires more flights in the expendable mode than the other LO_2/LH_2 Tugs studied by Mathematica in Phase I. The increased fees for this case are manifested in higher unit investment costs. For the other reusable Tugs, however, refurbishment cost variation is represented more closely by the user fee variation; and although Tug
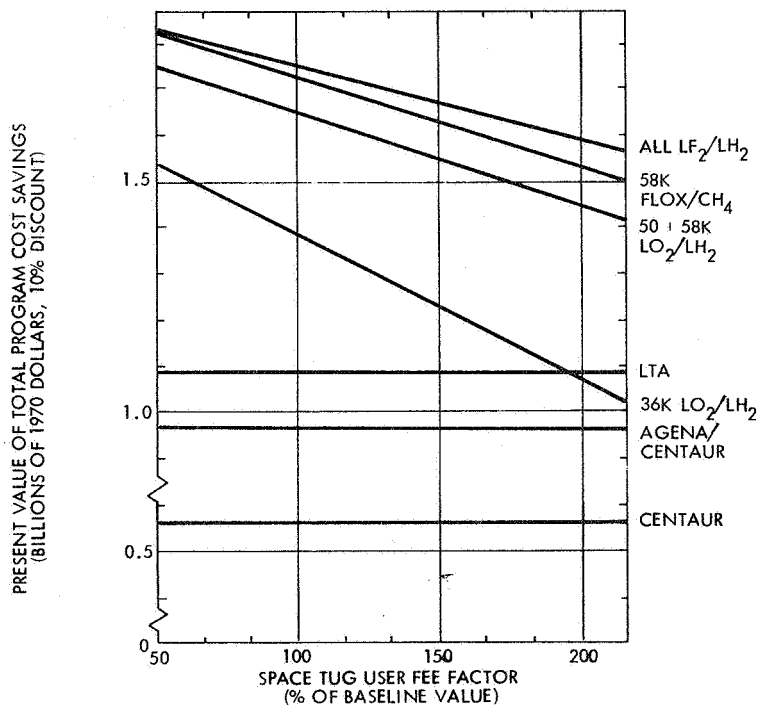
Figure 6-5 Phase I Sensitivity Analysis: Space Tug User Fee

user-fee increases tend to degrade the absolute cost effectiveness of the higher performing reusable Tugs, these Tugs remain unchanged on a relative basis.

Figure 6-6 illustrates the sensitivity of the various candidates to changes in payload refurbishment factors. Payload refurbishment factor is defined as the ratio of refurbishment cost to new unit cost, and the nominal study values range from 20 percent to 40 percent, with the majority between 30 and 40 percent. In fact, latest LMSC payload effects studies indicate that values around 20 percent may be realized. In Figure 6-6, the ordinate is defined as the present value of total program cost savings and, as explained above, is a measure of the cost effectiveness of the candidates in the study relative to the reference case. The payload refurbishment factors represented by the abscissa are given as percentages of the baseline values, i.e., 100 percent is equal to a nominal value of 20 percent to 40 percent, and 200 percent would be equivalent to a refurbishment factor of from 40 percent to 80 percent of payload unit cost.
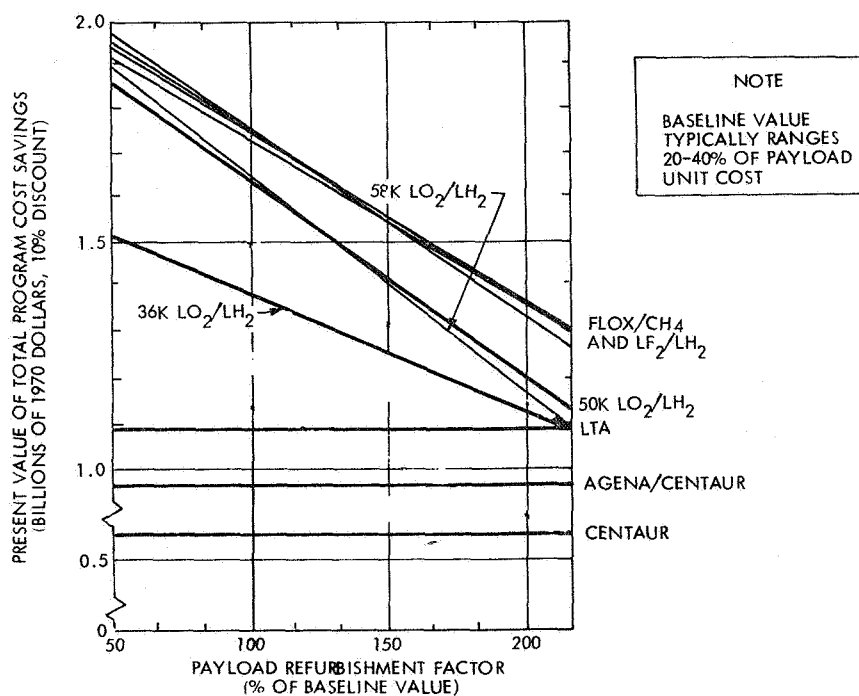
Figure 6-6 Phase I Sensitivity Analysis: Payload Refurbishment

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As expected, all reusable Tug candidates display extreme sensitivity to the payload refurbishment factors. This is because the major economic rationale for the Shuttle and Tug system is the lowering of space program cost through payload reuse. It is seen that the LO_2/LH_2 Tugs converge on the Large Tank Agena orbital injection stage at payload refurbishment costs of twice the nominal values. The fluorine-based Tugs, also highly sensitive to the payload refurbishment factor, remain somewhat more cost effective than the LO_2/LH_2 Tugs because of a slightly higher capture of potential mass and volume effects and lower transportation costs. If, in fact, the payload refurbishment factors do end up at the lower end of the range as indicated by the latest LMSC studies, the economic justification for a reusable Tug is strengthened.

Scenario Analysis

The way in which the choice of most cost-effective Tug or Tugs depends on the specific mission model used for the analysis is of major concern. The mission model is basically a projection of possible future activities based on current activity and experience. Although it is as good a planning document as is currently available, it cannot be said that the model will depict the actual programs of the 1980s and beyond with a high degree of confidence. Some mission model, however, must be used to perform the economic analysis. To overcome the limitations imposed by a fixed mission model, Mathematica developed a computer program, SCENARIO, that allows the user to perform economic analyses with parametric variations in the mission model. The mix of activity according to the user agency, the velocity level, and payload lifetime is variable in the analysis. These parameters may be varied singly or in combination. The scenarios presented in Table 6-3 cover a reasonable variation in the scale and composition of future unmanned space activity. Scenario 1 represents the baseline NASA-DoD mission model that was presented for purposes of study. The number of Shuttle and Tug flights required for the performance of the mission model is represented in Table 6-3 as 100 percent each. The actual flights for the Agena/Centaur family are 494 for the OIS and 507 for the Shuttle; for the 50,000 lb LO_2/LH_2 Tug there are 492 Tug and 562 Shuttle flights required. These vary according to the percentages given in Table 6-3.

Table 6-3 SPACE TUGS SCENARIO ANALYSIS VARIABLES

SCENARIO	DESCRIPTION	NO. OF SHUTTLE AND TUG FLIGHTS AS PERCENT OF BASELINE TRAFFIC	
		TUGS	SHUTTLES
1	BASELINE NASA-DoD MODEL	100	100
2	ALL 2 TO 4 YEAR SATELLITES REMOVED; 5 YEAR AND LONGER LIFETIME SATELLITES TRIPLED	101	99
3	OSS AND OA MODELS REDUCED TO 50%	79	80
4	DoD MODEL DOUBLED	132	134
5	OSS AND OA MODELS REDUCED TO 50% AND NON-NASA DOUBLED	105	105
6	SYNCHRONOUS EQUATORIAL MISSIONS DOUBLED	120	120
7	ALL 2 TO 4 YEAR SATELLITES REDUCED TO 75%; 5 YEAR AND LONGER SATELLITES INCREASED BY 50%	100	100

To determine the impact of varying payload lifetime, Scenario 2 considers the effect of removing all two to four year satellites in the mission model and tripling the number of five year and longer lifetime satellites. This represents approximately the same amount of Tug and Shuttle traffic as required by Scenario 1. Because of the rather high budget implications in the given mission model for the Office of Space Science and Applications (now the Office of Space Science and Office of Applications), there was interest in determining if there would be any impact on Tug choice should the activity level for this agency be half of what was given. Scenario 3 considers such an activity level; this scenario represents a reduction of Shuttle and Tug traffic to 80 percent and 79 percent of baseline values, respectively. Other scenarios include a doubling of the DoD traffic (Scenario 4), a reduction of the OSSA model to 50 percent and a doubling of non-NASA Applications (Scenario 5), a doubling of the synchronous equatorial missions (Scenario 6), and further variations in the mix of payload lifetimes in the mission model (Scenario 7). The range of activity resulting from these parametric variations was from 79 to 132 percent of the baseline Tug traffic and from 80 to 134 percent of the Shuttle traffic.

A major goal of the scenario analysis would be to determine whether the most important source of variation in Space Tug economic benefits lies with the scale of activity and not with the particular composition of space activity. If this is indeed found to be so, there is less concern with the fact that the given mission model is unlikely to be realized in exact measure over the 1980s. In other words, results are more certain when they depend on the activity level rather than the exact composition of that activity.

In the lower half of Figure 6-7, it is shown that based upon the reference case, (the Agena/Centaur family of orbit injection stages), losses can be expected if the Centaur is to be used alone. Because the observations from the scenarios fall on the benefits line, the losses that may be expected are proportional to the scale of activity. It seems that for this case it is indeed the scale of activity and not the composition that determines the economic results.

When payload mass and volume effects are introduced to the Centaur case (upper half of Figure 6-7), two observations representing Scenarios 2 and 5 fall away from the Centaur economic benefits line, with the observation for Scenario 2 lying considerably
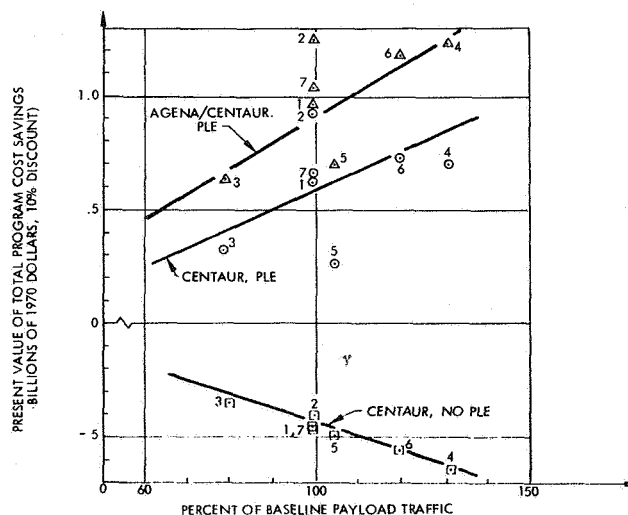

Figure 6-7 Space Tug Scenario Analysis: Existing Orbit Injection Stages

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above and the observation for Scenario 5 lying well below the line. Scenario 2, as shown in Table 6-3, represents an exercise in which all five-year payloads are tripled in number. The five-year payloads include Mission 35 which consists of 20 payloads with considerable mass-and-volume-related savings for Tugs with the required capabilities. It is not surprising therefore that a scenario that calls for a tripling of this mission would drive the economic benefits off the line of average relationship. Scenario 5, on the other hand, represents a doubling of non-NASA Applications missions and a halving of OSS and OA missions. The reduction in savings associated with this scenario is explained by the fact that the benefits for the non-NASA missions are lower than average because the RDT&E for these projected missions is assumed to have been accomplished in prior projects. Since the RDT&E savings via the payload effects are an important source of benefits, in particular for the larger capability Tugs, this has the effect of moving the results associated with Scenario 5 down from the benefits line.

Figure 6-8 shows the results of the scenario for the Large Tank Agena orbit injection stage. Its improvement economically over the Agena/Centaur family is related to reduced unit costs. Figures 6-9 and 6-10 show the results of the scenario analysis for the LO_2/LH_2, LF_2/LH_2, and $FLOX/CH_4$ reusable Space Tugs. With these Tug candidates, payload reuse becomes possible. It seems that while the observations for Scenario 5 continue to lie below the benefits line, the observations for Scenario 2 now also move below the line for the lower capability LO_2/LH_2 Tugs as mass and volume effects are reduced. On the other hand, as shown in Figure 6-9, the observation representing Scenario 4 lies above the benefits line, indicating larger than average payload reuse benefits for the DoD missions. Figure 6-10 displays the results of fluorine based propellant Tugs.

Figures 6-11 and 6-12 show the effects of a higher capability Shuttle and a lower capability Shuttle. It is seen in Figure 6-11 that the higher capability Shuttle produces hardly any impact on the economic benefits derived from the 58,000 lb LO_2/LH_2 Tug. In Figure 6-12, it is seen that a lower capability Shuttle apparently has a degrading impact on the cost effectiveness of the 36,000 lb LO_2/LH_2 Tug for some scenarios, with potential savings reduced by 25 to 30 percent. Figure 6-13 is a summary of the

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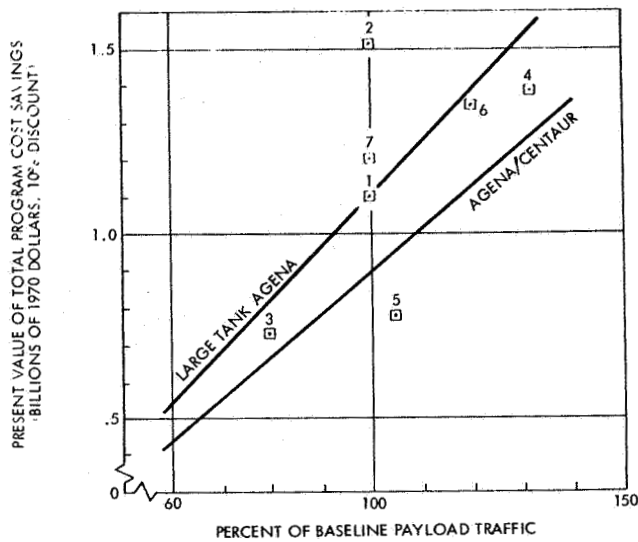
Figure 6-8 Space Tug Scenario
Analysis: Large
Tank Agena
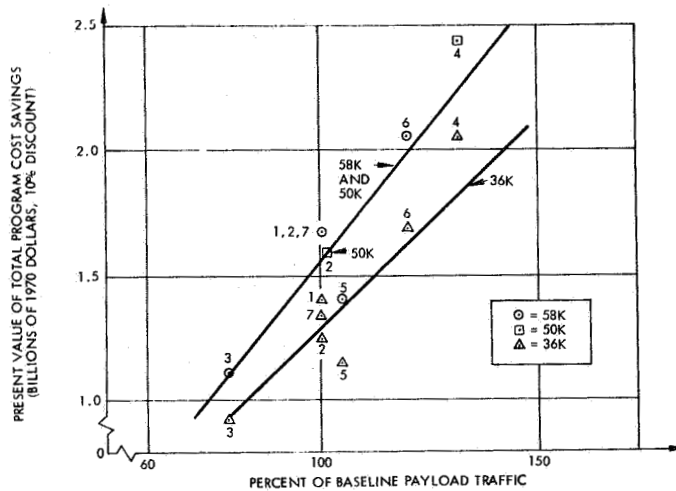
Figure 6-9 Space Tug Scenario
Analysis: LO_2/LH_2
Tugs
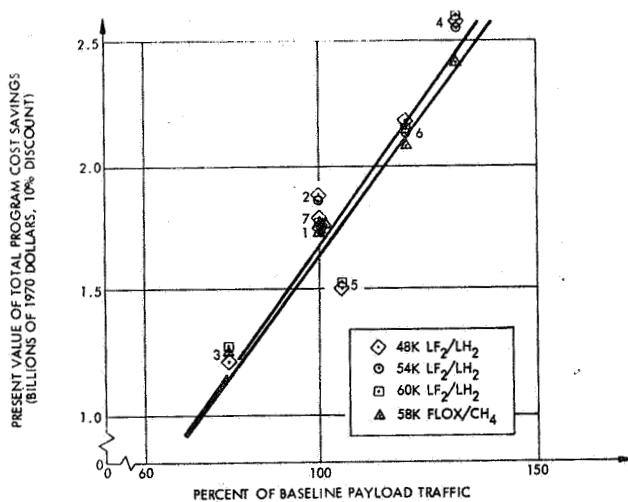
Figure 6-10 Space Tug Scenario
Analysis: LF_2/LH_2
and FLOX/CH_4
Tugs
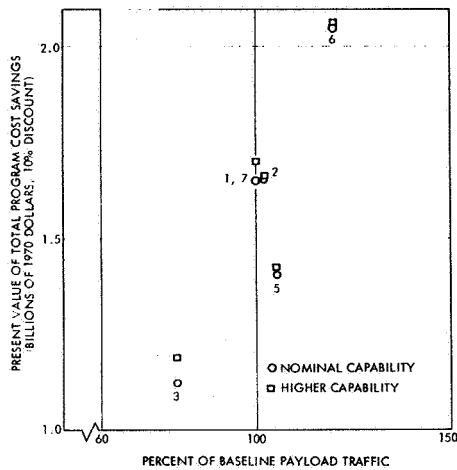
Figure 6-11 Space Tug Scenario Analysis: 58K LO_2/LH_2 Tug, Nominal vs Higher Capability Shuttle
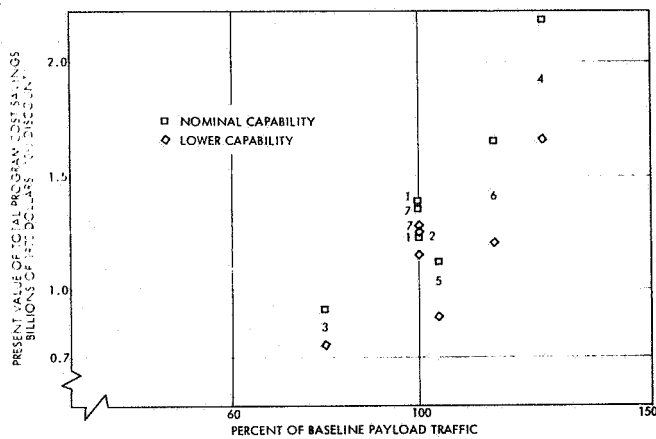
Figure 6-12 Space Tug Scenario Analysis: 36K LO_2/LH_2 Tug, Nominal vs Lower Capability Shuttle
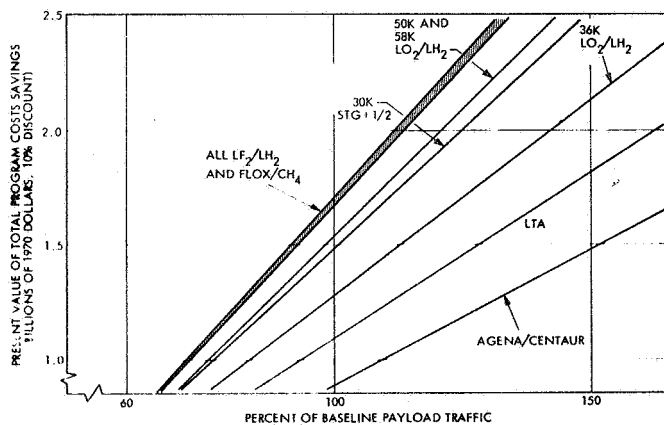
Figure 6-13 Space Tug Scenario Analysis: Summary of Tug Candidates

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results of the scenario analyses performed for the orbit injection stages and reusable Tug candidates. It is seen that the larger capability 50,000 lb and 58,000 lb LO_2/LH_2 Tugs and the fluorine based Tug candidates provide economic benefits of approximately $1 billion even with the activity level reduced to 70 or 75 percent of the nominal activity level. It is also seen that the increase in benefits with activity level is generally uniform and is predictably steeper with the higher capability reusable Tugs.

Summary of Benefits

We now turn to a summary of the Tug economic benefits as derived in the Phase I economic analysis (Figures 6-14 through 6-17.) In Figure 6-14 there are two vertical axes. The left axis is the present value of recurring cost savings, that is, the savings associated with payload reuse, mass and volume effects, and lower cost transportation. Not included in the recurring costs are RDT&E and, in the case of the reusable Tugs, the Tug fleet investment. The right vertical axis shows the conversion of the recurring cost savings into allowable nonrecurring cost. At any given discount rate, in this case 10 percent, it is possible to convert the recurring cost savings into an equivalent economic value that is a function of the discount rate, the time, and the spreading functions at which the RDT&E and fleet investment costs are incurred. Put in another way, the allowable nonrecurring cost tells us how much, at a given discount rate, we could spend on nonrecurring expenditures (not would like to spend or necessarily would spend) and be economically indifferent between the Tug in question and the reference case. It is seen in Figure 6-14 that given the potential savings from mass and volume effects, payload reuse, and lower cost transportation, up to $4 billion could in theory be spent to develop and produce the higher capability Space Tugs and yield a cost effective investment at 10 percent. This value is a hypothetical saving since every candidate Tug can capture some mass and volume savings. Nonetheless, this result underlies the importance of the contribution of a Space Tug in the Shuttle/Tug transportation system. As shown in Figure 6-14 the benefits from either the NASA unmanned missions or the DoD missions alone justify the development and investment in a Space Tug system.

(Billions of 1970 Dollars, 10%)
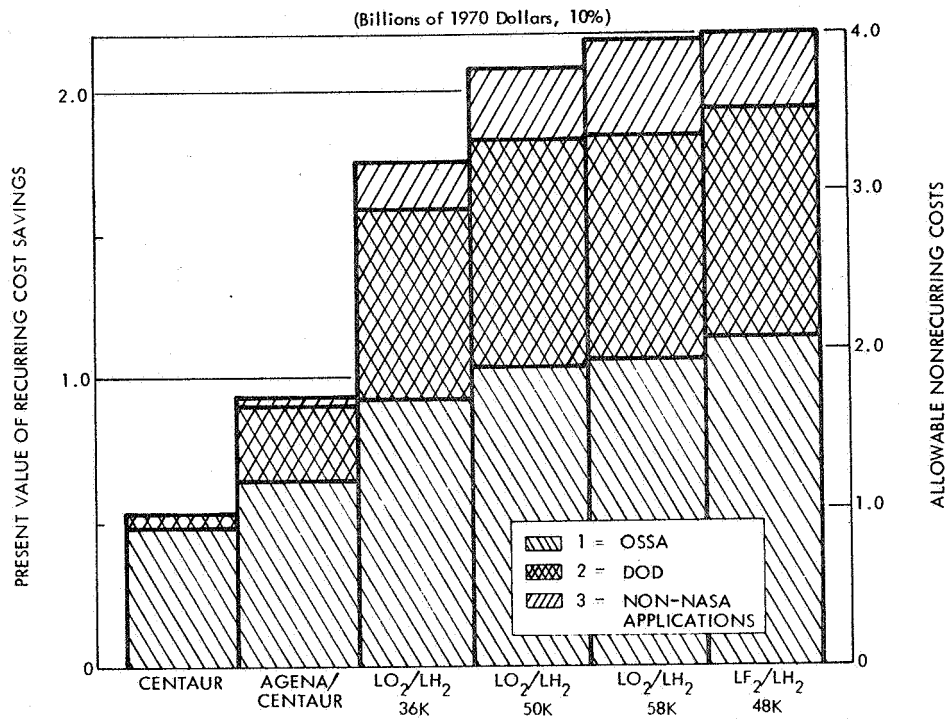
Figure 6-14 Benefits by Agency (Phase I)

(Billions of 1970 Dollars, 10%)
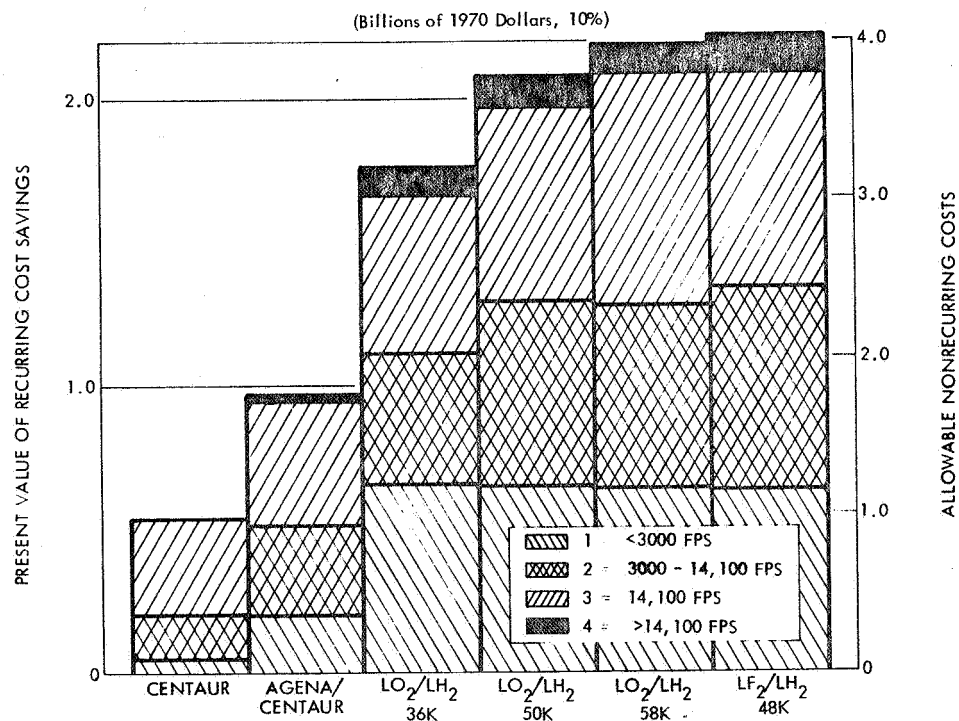
Figure 6-15 Benefits by Velocity Level (Phase I)

(Billions of 1970 Dollars)

μ_H 1.234

σ_H 0.144

0.5   0.6   0.7   0.8   0.9   1.0   1.1   1.2   1.3   1.4   1.5   1.6   1.7   1.8   1.9

UPPER UNCERTAINTY FACTORS APPLIED

$\mu_L = 0.832$

$\sigma_L = 0.080$

0.5   0.6   0.7   0.8   0.9   1.0   1.1   1.2   1.3   1.4   1.5   1.6   1.7   1.8   1.9

LOWER UNCERTAINTY FACTORS APPLIED

$\mu_N = 0.867$

$\sigma_N = 0.076$

0.5   0.6   0.7   0.8   0.9   1.0   1.1   1.2   1.3   1.4   1.5   1.6   1.7   1.8   1.9

NOMINAL VALUES APPLIED
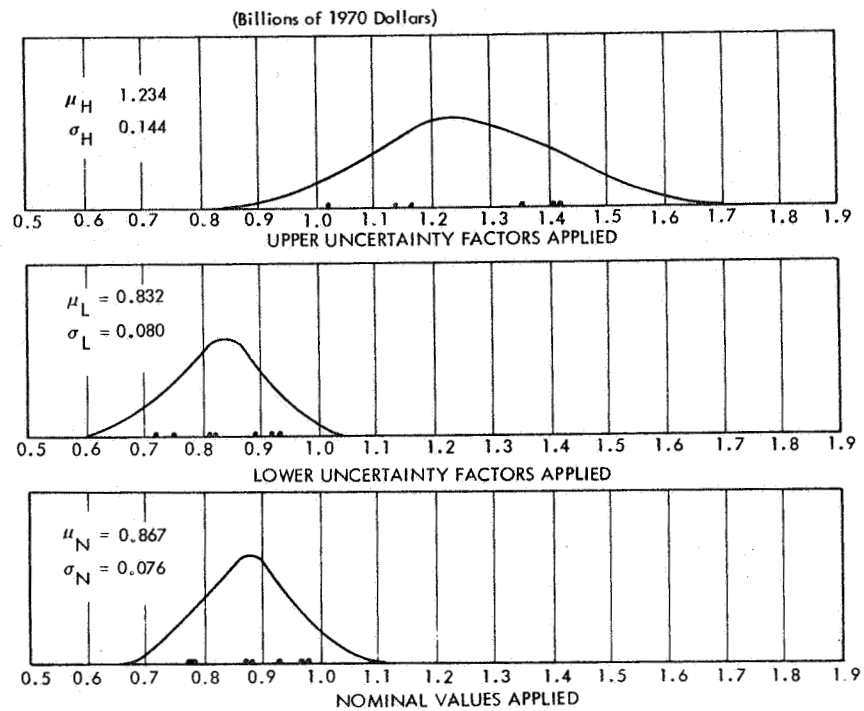
Figure 6-16   Application of Uncertainty Factors

$\mu_4 = 5.105$

$\sigma_4 = 0.718$

$\mu_1 = 3.817$

$\sigma_1 = 0.399$

$\mu_3 = 2.911$

$\sigma_3 = 0.326$

ALLOWABLE NONRECURRING COSTS (BILLIONS OF 1970 DOLLARS)

ESTIMATED NONRECURRING COSTS BASED ON HIGHER UNCERTAINTY FACTORS

60   100   150

PAYLOAD TRAFFIC AS PERCENT OF BASELINE
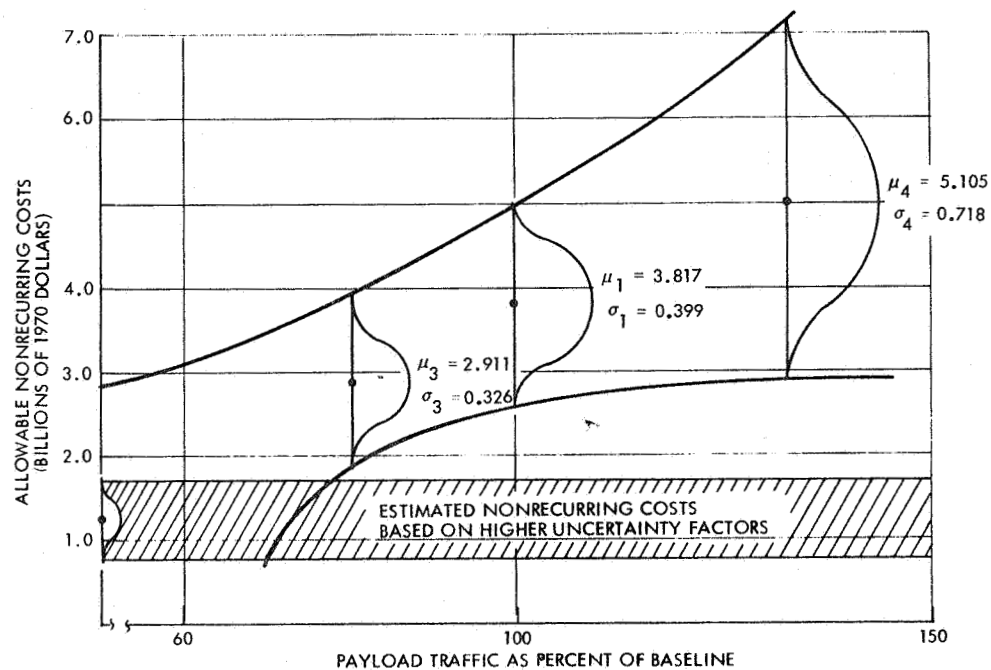
Figure 6-17   Combined Effects of Scenario Analyses and Uncertainties

6-18

Although the benefits associated with non-NASA applications missions appear relatively small, it should be recalled that these missions represent only a small percentage of the activity in the total mission models, and therefore the benefits associated with these programs are roughly proportionately equal to those accruing to the other using agencies.

Figure 6-15 shows a very important result: that the economics of a reusable Space Tug do not depend upon payload reuse at synchronous equatorial orbits. As shown, about $2.5 billion in allowable nonrecurring cost is associated with orbits less than synchronous equatorial. About $1.2 billion in allowable nonrecurring (more than the expected cost of developing and producing a reusable Space Tug) are associated with orbits less than 3000 feet per second. It is also seen that in the synchronous equatorial regime approximately two-thirds of the benefits are derived from mass and volume effects, not payload reuse. For the final assessment of Tug benefits see Chapter 7.

Figure 6-16 shows the mean, standard deviation, and 3-sigma distribution of the reusable Tug nonrecurring costs when the uncertainty factors given in Table 6-2 are applied to the nonrecurring costs. Figure 6-17 shows the mean and 3-sigma distribution of the recurring cost savings associated with various Tug candidates as derived from the scenario analysis. Based upon the 3-sigma distribution of allowable nonrecurring cost derived from Scenarios 1, 3, and 4, and the 3-sigma estimate of nonrecurring cost, it may be stated that assuming the most pessimistic expectation of costs and benefits, and payload traffic as low as 75 percent of the baseline NASA/DoD high energy model, a reusable Space Tug will be cost-effective evaluated at the 10 percent rate of discount.

Figure 6-18 addresses the question of the reusable Tug Initial Operating Capability (IOC) date. Given the rapid build up of benefits (cost savings) based upon the Agena/ Centaur case without payload effects calculated on this figure for the 1973-1983 period, a concurrent IOC of a reusable Tug with the Shuttle appears desirable. However, it is acknowledged that conditions could prevail, such as a Shuttle phase-in and/or NASA budget constraints, that could alter this result.
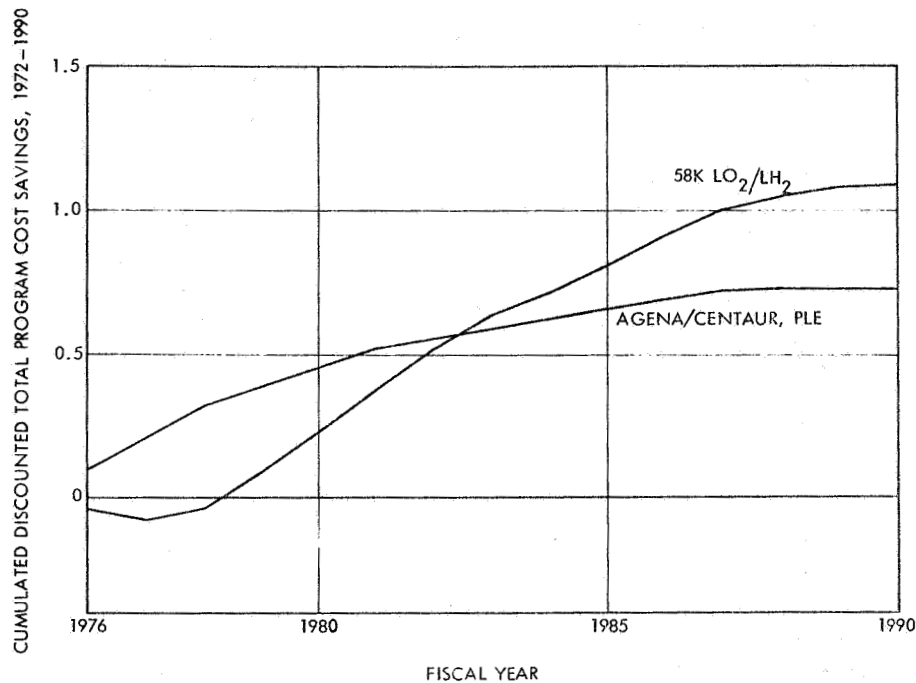
Figure 6-18 Time Phasing of Tug Benefits

Phase I Economic Conclusions

In light of the economic analysis presented above, the following conclusions pertain to Phase I analyses:

1. A reusable Space Tug is a cost-effective investment evaluated at the 10 percent discount rate.

2. A reusable Tug IOC coincident with the Space Shuttle IOC appears desirable.

3. Among the LO_2/LH_2 candidates, the 50,000 lb Tug is least sensitive to recurring program cost uncertainties and the 36,000 lb Tug is the most sensitive.

4. The LF_2/LH_2 and $FLOX/CH_4$ Tugs provide the highest nominal savings and are relatively insensitive to recurring cost sensitivities; however, their potential advantages could be diminished because of nonrecurring cost uncertainties.

5. The scenario analysis indicates that it is activity level, not mission model composition that determines Space Tug benefits for a given mission model.

Chapter 7

PHASE II ECONOMIC ANALYSIS

The efforts in Phase II represent a refinement and extension of the analysis conducted by Mathematica during Phase I. The objective of Phase I was to narrow down the choice of Tug and orbit injection stage candidates to bring into the Phase II analysis those that were likely choices for future development. Also in Phase II some additional candidates that had not yet been analyzed by Mathematica, such as the 30,000 lb $LO_2/$ LH_2 stage-and-one-half concept, the 30,000 lb LO_2/LH_2 single-stage Tug, and some families of expendable and reusable stages were to be studied.

Figure 7-1 presents the nominal economic values for the principal candidates studied in Phase II. This figure is constructed in the same way as Figure 6-1 in Phase I, except that in Phase II the set of candidates was reduced and the reference case was changed to the Large Tank Agena orbital injection stage with assumed mass–and–volume payload effects. Because of the change in reference cases, the economic benefits accruing from the candidate Tugs in Phase II are derived primarily from payload reuse, and to a lesser extent, lower cost transportation. As seen in Figure 7-1, the Large Tank Agena, the new reference case, foregoes approximately $1 billion in recurring cost saving opportunities. The maximum savings is realized by the 48,000 lb LF_2/LH_2 candidate. Figure 7-1 also shows that the mass and volume effects represent about $1 billion in recurring cost savings. This is indicated by the difference between the recurring cost savings associated with candidate 1, the Agena/Centaur family without payload effects, and candidate 2 the Agena/Centaur with payload effects. As we move from the Agena/Centaur with payload effects to the Large Tank Agena, there are some additional savings because of mass and volume effects and RDT&E costs.

In Figure 7-1 it is shown that the 30,000 lb LO_2/LH_2 Tug is barely cost effective with respect to the Large Tank Agena reference case, but that as the reusable Tugs increase their payload retrieval capability, their cost effectiveness increases. The 48,000 lb

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BILLIONS OF 1970 DOLLARS, 10%

1 - Agena/Centaur, No PLE
2 - Agena/Centaur, PLE
3 - Large Tank Agena, PLE
4 - 30K LO_2/LH_2, PLE
5 - 36K LO_2/LH_2, PLE
6 - 30K LO_2/LH_2, STG + 1/2, PLE
7 - 50K LO_2/LH_2, PLE
8 - 48K LF_2/LH_2, PLE
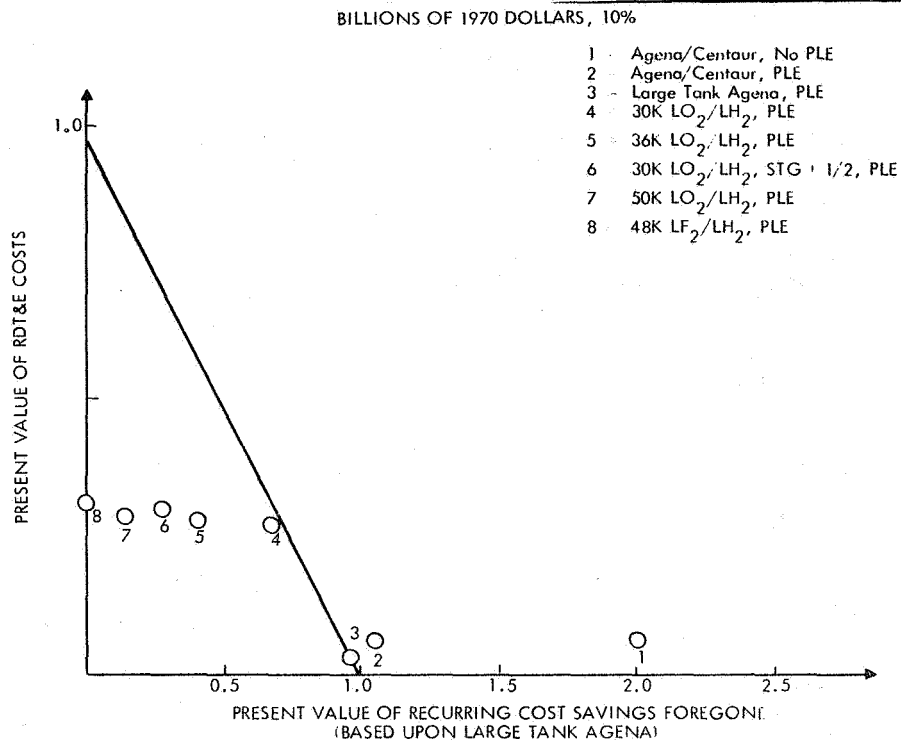
Figure 7-1 Recurring vs Nonrecurring Costs

LF_2/LH_2 Tug was brought into Phase II as being the best candidate among the fluorine-based-propellant Tugs. The 50,000 lb LO_2/LH_2 Tug represented the most favorable member in its particular class. The stage-and-one-half and 30,000 lb LO_2/LH_2 Tugs were first introduced into the economic analysis during Phase II, and the 36,000 lb LO_2/LH_2 Tug was kept in the study because of its adaptability to shortened Shuttle payload bays.

Effects of Uncertainties

Figure 7-2 shows the eight candidates with the higher, nominal, and lower Tug RDT&E cost uncertainty bounds as given in Table 6-2 applied to them. As expected, the RDT&E uncertainty factors impact the 48,000 lb LF_2/LH_2 candidate the most. Yet, as shown, even with the uncertainty factors considered, this Tug remains the most cost effective of the various candidates.

BILLIONS OF 1970 DOLLARS, 10%
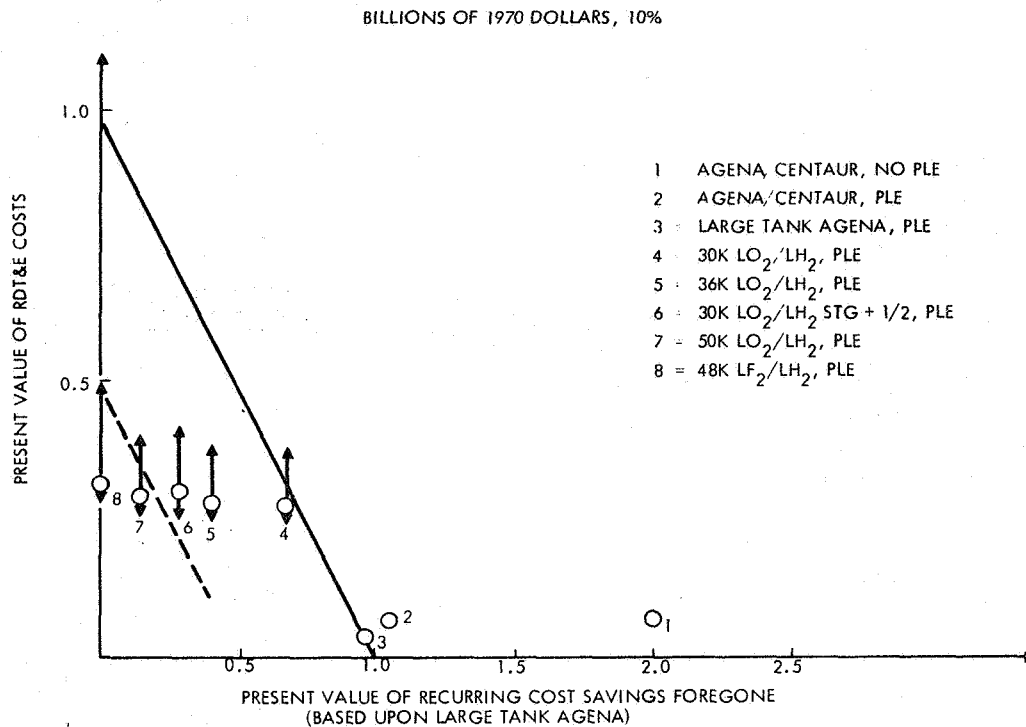
Figure 7-2 Recurring vs Nonrecurring Costs With Tug RDT&E Uncertainty Factors

Figure 7-3 is concerned with the impact of payload uncertainty factors in the Phase II analysis. As payload cost, especially unit and investment cost, are driven upward by the uncertainty factors, the benefits from payload reuse are increased and greater losses are incurred from expendable mode operations. This is demonstrated in the figure with the reference Large Tank Agena orbit injection stage foregoing an additional $150 million in present values, whereas the 36,000 lb LO_2/LH_2 Tug loses only about $50 million in present values and the 48,000 lb LF_2/LH_2 and 50,000 lb LO_2/LH_2 Tugs remain relatively unchanged.

Economic Sensitivities

Figure 7-4 addresses the impact of the Space Shuttle user fee as a variable in the Phase II analysis. Figure 6-4 of Phase I has been updated to include the two additional candidates, the 30,000 lb LO_2/LH_2 single-stage Tug and the stage-and-one-half concept. As shown in Figure 7-4, the stage-and-one-half Tug is highly sensitive to the Space Shuttle user fee. This is because 125 more Shuttle flights are required for this

BILLIONS OF 1970 DOLLARS, 10%

PAYLOAD DEFINITION FACTOR	
GOOD	1.10
FAIR	1.20
POOR	1.30

1 — AGENA/CENTAUR, NO PLE
2 — AGENA/CENTAUR, PLE
3 — LARGE TANK AGENA, PLE
4 — 30K LO_2/LH_2, PLE
5 — 36K LO_2/LH_2, PLE
6 — 30K LO_2/LH_2, STG + 1/2, PLE
7 — 50K LO_2/LH_2, PLE
8 — 48K LF_2/LH_2, PLE

PRESENT VALUE OF RDT&E COSTS

PRESENT VALUE OF RECURRING COSTS FOREGONE
(BASED UPON LARGE TANK AGENA)
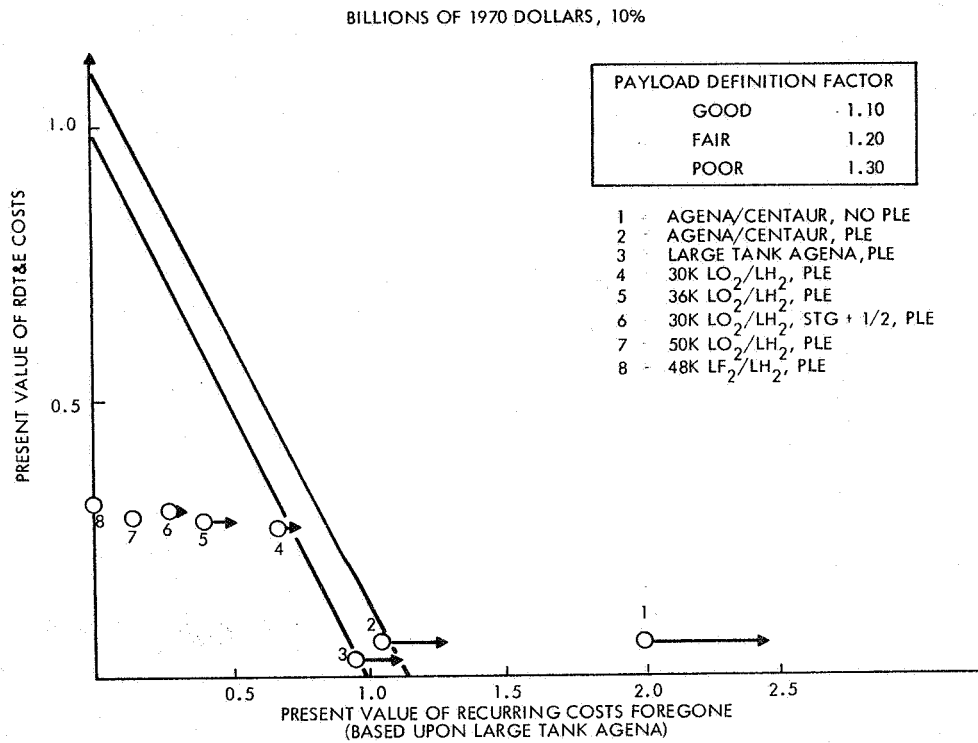
Figure 7-3 Recurring vs Nonrecurring Costs with Payload Uncertainty Factors

PRESENT VALUE OF TOTAL PROGRAM COST SAVINGS
(BILLIONS OF 1970 DOLLARS, 10% DISCOUNT)

48K LF_2/LH_2
58K FLOX/CH_4
54 + 60K LF_2/LH_2
50K LO_2/LH_2
58K LO_2/LH_2
36K LO_2/LH_2
30K LO_2/LH_2 STG + 1/2
30K LO_2/LH_2
LTA
AGENA/CENTAUR
CENTAUR

SHUTTLE USER FEE
(MILLIONS OF 1970 DOLLARS)
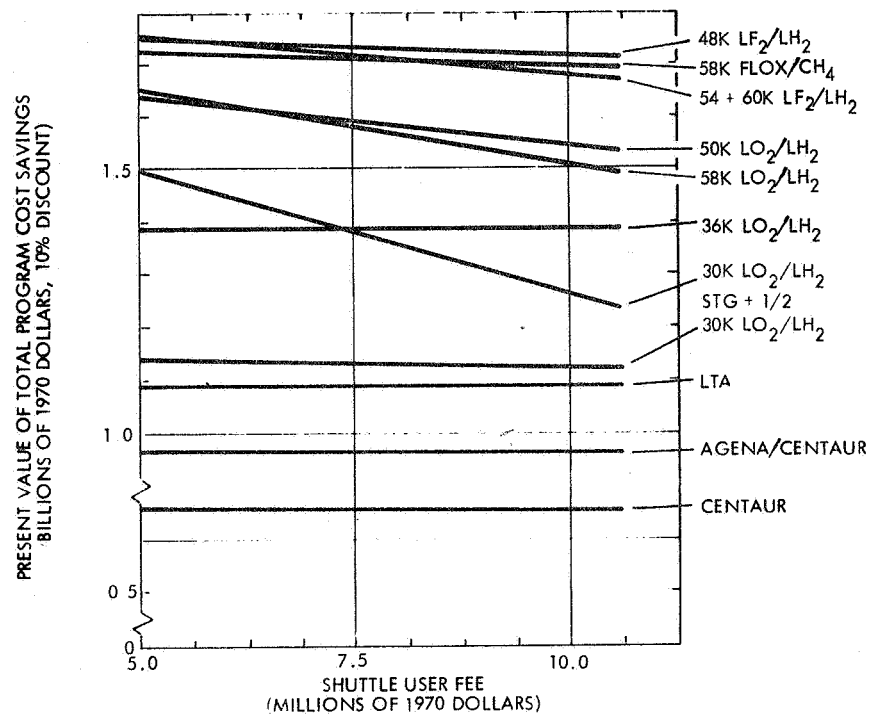
Figure 7-4 Sensitivity Analysis: Space Shuttle User Fee

7-4

configuration than for the reference case. Also as shown, the 30,000 lb LO_2/LH_2 single-stage Tug is relatively insensitive to the Shuttle user fee because it requires approximately the same number of Shuttle flights as the reference case.

Figure 7-5 shows the impact of the payload refurbishment, and is another update of a figure used in Phase I. The 30,000 lb single-stage concept and the stage-and-one-half Tug behave in fashion similar to the other LO_2/LH_2 candidates, but the 30,000 lb single-stage Tug cost effectiveness deteriorates very rapidly as the payload refurbishment factor increases. This is explained by the fact that the cost savings that would be provided by this Tug depend primarily upon payload reuse benefits, because it does not have the payload capability to capture significant mass and volume effects. Again, the latest LMSC payload effects study indicates that we may expect the payload refurbishment factor to tend toward the lower end of the indicated range, which of course improves the economic values for all of the reusable Tug candidates.
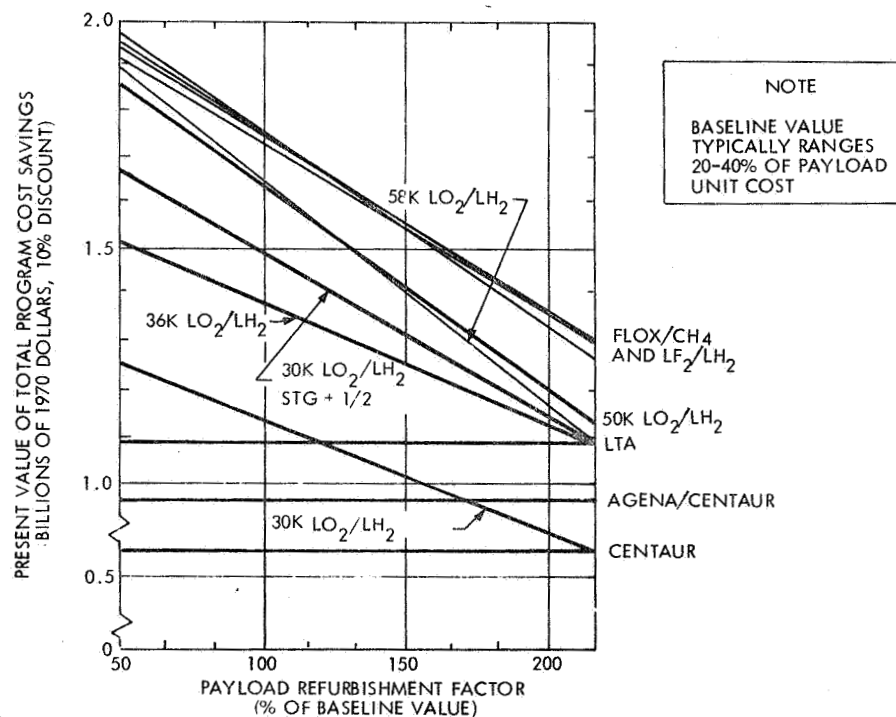
Figure 7-5 Sensitivity Analysis: Payload Refurbishment

7-5

Tug Benefits Analysis

The allowable RDT&E cost is the expenditure (in undiscounted dollars), up to and including which, costs may be incurred for Tug RDT&E and still have the candidate concept be cost effective when the investment is evaluated at the 10 percent rate of discount. The allowable RDT&E is a function of the recurring cost savings, the discount rate, and the time spread over which the RDT&E costs are incurred. Of course, we do not expect to spend the total allowable RDT&E on the Tug candidate; and the difference between the total allowable cost and the estimated RDT&E cost is considered to be the economic margin associated with each Tug candidate. This economic margin provides an insight into: (1) the margin to cover error in the estimation of costs and benefits, and (2) the return, over its cost measured in undiscounted 1970 dollars, to the Shuttle/Tug space transportation system. Table 7-1 shows that without a reusable Space Tug in the new transportation system, the Space Shuttle could lose over $1 billion in potential allowable nonrecurring cost. Also shown in Table 7-1 are some initial

Table 7-1 ALLOWABLE RDT&E COST AND SENSITIVITY SUMMARY
MILLIONS OF 1970 DOLLARS, UNDISCOUNTED

TUG CONFIGURATION	COMPUTED ALLOWABLE RDT&E COST*	ESTIMATED RDT&E COST	ECONOMIC MARGIN	Δ ALLOWABLE RDT&E COST	
				PER .01 INCREASE IN PAY-LOAD p	PER $1M INCREASE IN SHUTTLE USER FEE
30K LO_2/LH_2	574	502	72	-4	-5
36K LO_2/LH_2	1062	510	552	-5	-5
50K LO_2/LH_2	1554	528	1026	-8	-42
48K LF_2/LH_2	1809	576	1233	-7	-22
FAMILY: 30K LO_2/LH_2 + DROP TANKS	1296	551	745	-6	-90
FAMILY: 20K & 50K LO_2/LH_2	1819	647	1172	-	-
FAMILY: 20K LO_2/LH_2 + DROP TANKS	1335	533	802	-	-
FAMILY: 20K LO_2/LH_2 + LTA	828	539	289	-	-
SPACE BASED (CASE 3)	2227	569	1658	-	-

*BASED ON LTA AS REFERENCE CASE; TUGS TANDEM IN MODES 2 AND 4 ONLY

results for some families of Tugs which include the 20,000 lb and 50,000 lb LO_2/LH_2 concepts (with shared RDT&E costs), a 20,000 lb single stage with expendable drop tanks, and a 20,000 lb LO_2/LH_2 reusable Tug plus a Large Tank Agena OIS. These cases have been presented in order to round out the study, but it should be noted that they have not been analyzed to the extent that the others have, and hence the results should be regarded as initial. The emphasis for this summary is on those cases that have been fully analyzed.

In Table 7-1, sensitivities of the allowable RDT&E costs to changes in the payload refurbishment factor and Shuttle user fee are given. As shown, a change in payload refurbishment factor from, say, 30 to 31 percent for the 50,000 lb LO_2/LH_2 Tug reduces its allowable RDT&E by $8 million. The most significant sensitivity displayed in this table is the decrease in allowable RDT&E costs associated with a $1 million increase in Shuttle user fee. As we have already seen in Figure 7-4, the stage-and-one-half candidate is highly sensitive to this variable, with a $1 million increase in Shuttle user fee reducing the allowable RDT&E cost by $90 million.

Table 7-2 summarizes the variation in allowable RDT&E cost with respect to variations in Tug specific impulse. From this table, it is seen that there is a diminished specific impulse sensitivity as propellant weights rise. The numbers in the parentheses indicate the differences between the nominal case – the 460 sec specific impulse engine – and the alternatives, the 444 sec specific impulse RL10 engine and the 470 sec specific impulse engine. As is seen in Table 7-2, the economic margin of the 444 sec RL10 engine is greater than that of the 460 sec baseline engine and hence the RL10 may be considered an excellent engine selection, especially in view of the minimal development risk and cost uncertainty.

Table 7-3 summarizes the allowable RDT&E cost results for selected Tugs based upon a sensitivity analysis of the stage mass fractions. This sensitivity analysis indicates that there exists a potential of extreme variation in the benefits associated with the LO_2/LH_2 Space Tugs, with a possible cost ineffectiveness (evaluated at 10 percent) indicated for the smaller propellant weights. Moreover, it appears that unwillingness to invest more money to increase the mass fraction of the LO_2/LH_2 Tugs might make it desirable to invest a fair amount to hedge against mass fraction reduction. The

Table 7-2 SPECIFIC IMPULSE SENSITIVITY ANALYSIS

MILLIONS OF 1970 DOLLARS, UNDISCOUNTED

TUG CANDIDATE	I_{SP} (SEC)	COMPUTED ALLOWABLE RDT&E COST* (AND Δ COST)	ESTIMATED RDT&E COST	ECONOMIC MARGIN
36K LO_2/LH_2	444	857 (−205)	441	416
	460	1062	510	552
	470	1274 (+212)	530	744
50K LO_2/LH_2	444	1507 (−47)	456	1051
	460	1554	528	1026
	470	1636 (+82)	548	1088
58K LO_2/LH_2	444	1537 (−33)	469	1068
	460	1570	537	1033
	470	1614 (+44)	558	1056

*BASED ON LTA AS REFERENCE CASE; TUGS TANDEM IN MODES 2 AND 4 ONLY

Table 7-3 MASS FRACTION SENSITIVITY ANALYSIS

MILLIONS OF 1970 DOLLARS, UNDISCOUNTED

TUG CANDIDATE	VARIATION	COMPUTED ALLOWABLE RDT&E COST* (AND Δ COST)	ESTIMATED RDT&E COST**	ECONOMIC MARGIN
36K LO_2/LH_2	0.832	1,879 (+817)	510	1,369
	0.842	1,381 (+319)	510	871
	0.852 (NOM)	1,062	510	552
	0.862	673 (−389)	510	163
	0.872	−98 (−1160)	510	−608
50K LO_2/LH_2	0.853	1,788 (+234)	528	1,260
	0.863	1,712 (+158)	528	1,184
	0.873 (NOM)	1,554	528	1,026
	0.883	1,327 (−227)	528	799
	0.893	931 (−623)	528	403
48K LF_2/LH_2	0.862	1,911 (+102)	576	1,335
	0.872	1,845 (+36)	576	1,269
	0.882 (NOM)	1,809	576	1,233
	0.892	1,783 (−26)	576	1,207
	0.902	1,575 (−234)	576	999

*BASED ON LTA AS REFERENCE CASE; TUGS TANDEM IN MODES 2 AND 4 ONLY
**NOMINAL FOR GIVEN TUG

48,000 lb LF_2/LH_2 Tug, on the other hand, is relatively insensitive to changes in the value of the mass fraction parameter. For all of these Tugs, no attempt was made to estimate variations in expected RDT&E cost associated with varying mass fraction because the data base is insufficient to estimate the incremental RDT&E cost to change Tug mass fraction by one or two points. Hence, it is not possible to calculate the changes in economic margin for variations in the mass fraction.

Table 7-4 summarizes allowable RDT&E costs for the 50,000 lb LO_2/LH_2 Tug with respect to variations in lifetime, first unit cost, and refurbishment-factor variables. The table shows that of the three parameters investigated, our major concern is with the impact of Tug use life on the cost effectiveness of a reusable Tug. As in the case of mass fraction, while it appears that it may not pay to incur higher unit costs or refurbishment costs to attain greater Tug use ife, there should be willingness from the risk standpoint to incur these higher costs to avoid a reduction in lifetime. Once again, the estimated RDT&E costs shown here are nominal values and do not include the as-yet unknown development costs to improve refurbishment and lifetime.

Table 7-4 TUG LIFETIME AND REFURBISHMENT SENSITIVITY ANALYSIS

Lifetime (No. of Uses)	Refurbishment Factor (%)	First Unit Cost	Computed Allowable RDT&E Cost* (and ΔCost)	Estimated RDT&E Cost**	Economic Margin
100	3	16.4	1,808 (+254)	528	1,280
100	6	16.4	1,632 (+78)	528	1,104
100	3	32.8	1,446 (−108)	528	918
30	10	16.4	1,184 (−370)	528	656
30	6	16.4	1,377 (−177)	528	849
30	3	16.4	1,554 (Nominal)	528	1,026
30	2	16.4	1,607 (+53)	528	1,079
30	1	16.4	1,670 (+116)	528	1,142
10	3	16.4	831 (−723)	528	303
10	1	16.4	928 (−626)	528	400

*Based on LTA as reference cost; Tugs tandem in all modes
**Nominal for this Tug

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Figure 7-6 comprises two independent graphs, one a plot of the allowable RDT&E cost associated with variations in Tug lifetime holding refurbishment constant, and the other a plot of refurbishment factor holding use life constant to show the trend in these factors as Tug design improves. The figure indicates diminishing economic returns as lifetime passes 30 uses, while a relatively constant improvement economically is shown with respect to variation in the refurbishment factor between 10 and 1 percent.

Figure 7-7 indicates the sources of economic benefits for the Tug candidates with respect to the various system elements. The right vertical scale shows the conversion of recurring cost savings into allowable RDT&E costs. These benefits and allowable costs are divided by the major system elements, i.e., transportation and payloads. Given the potential savings from payload reuse and lower transportation costs (almost all payload and mass and volume effects are achieved by the reference case, the Large Tank Agena) when evaluated at the 10 percent discount rate, up to $1.75 billion (undiscounted) could be spent to develop the higher capability reusable Space Tugs. Transportation cost savings, alone, do not justify the development of a reusable Space Tug.

Figure 7-8 shows the distribution of expected benefits among the user agencies. For the most cost-effective Tugs, the savings derived from payload reuse and reduced transportation cost justified the Space Tug RDT&E cost for each of the agencies.

Figure 7-9 illustrates that the largest savings by program area accrue primarily to the earth observations program and secondarily to the communications programs. This result is independent of the agency performing these classes of missions. The values given in Figure 7-10 show the distribution of the largest category of benefits — those associated with the earth observation program — by agency and velocity level. The benefits are distributed quite equally among the various candidates both by agency and velocity level. Most important, the benefits by velocity level are concentrated in orbits less than synchronous equatorial.
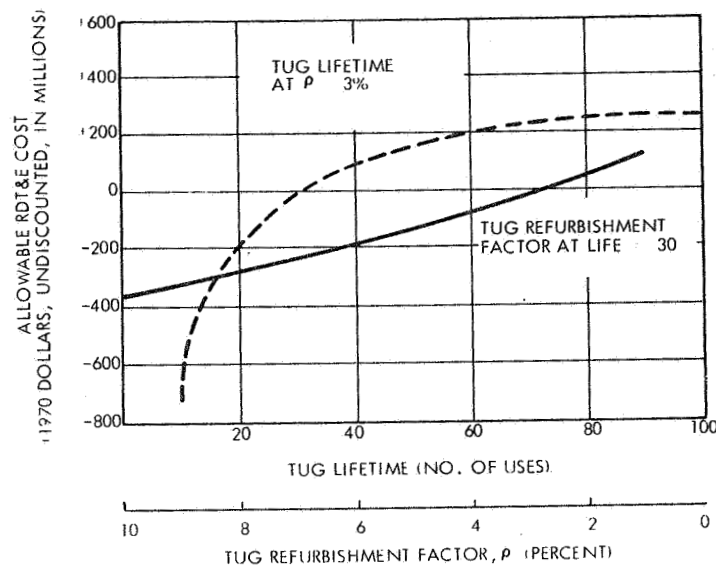
Figure 7-6 Allowable RDT&E vs Lifetime and Refurbishment Factors
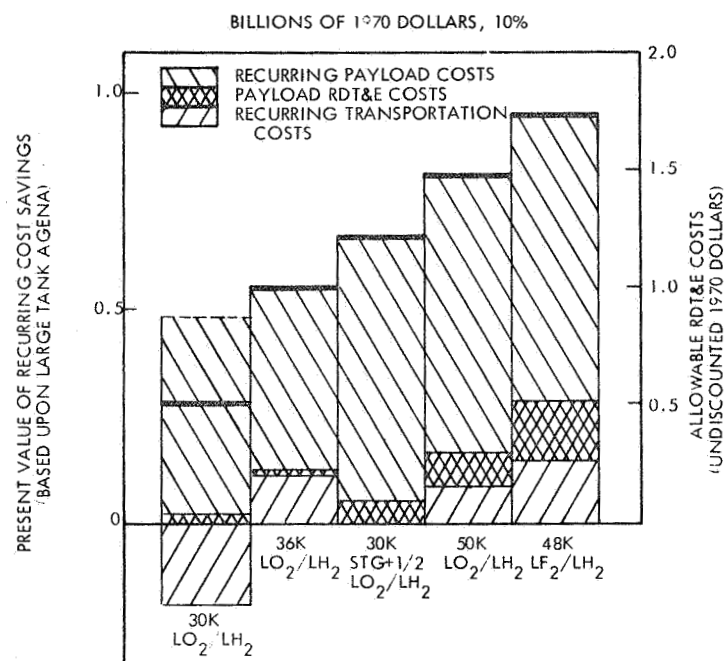
Figure 7-7 Benefits by Agency

7-11
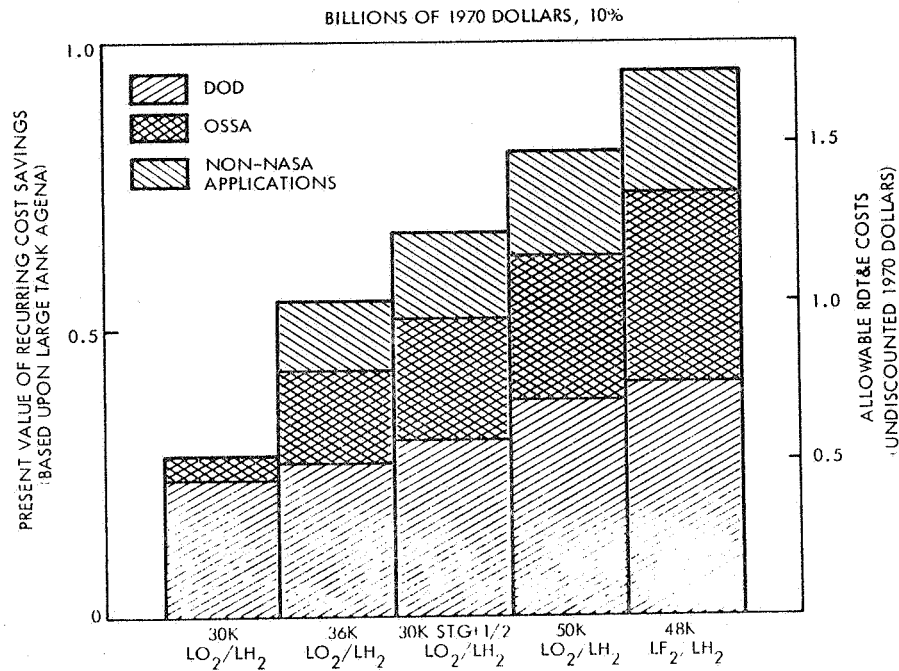
Figure 7-8 Benefits by System Element
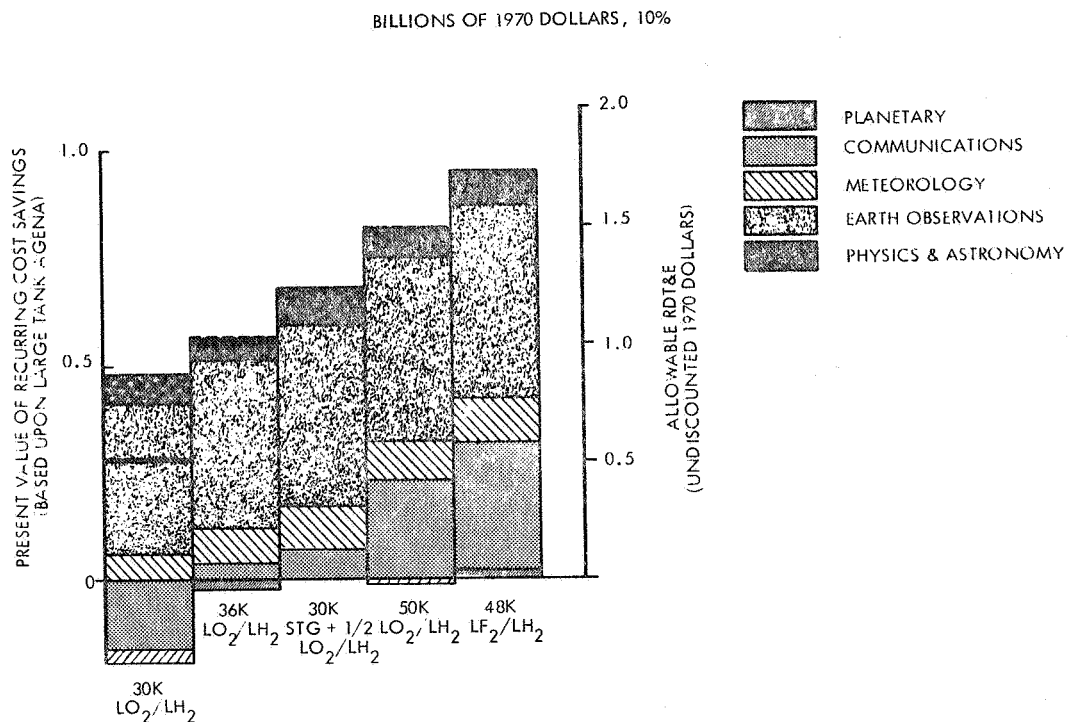
Figure 7-9 Benefits by Program Area

BILLIONS OF 1970 DOLLARS, 10%
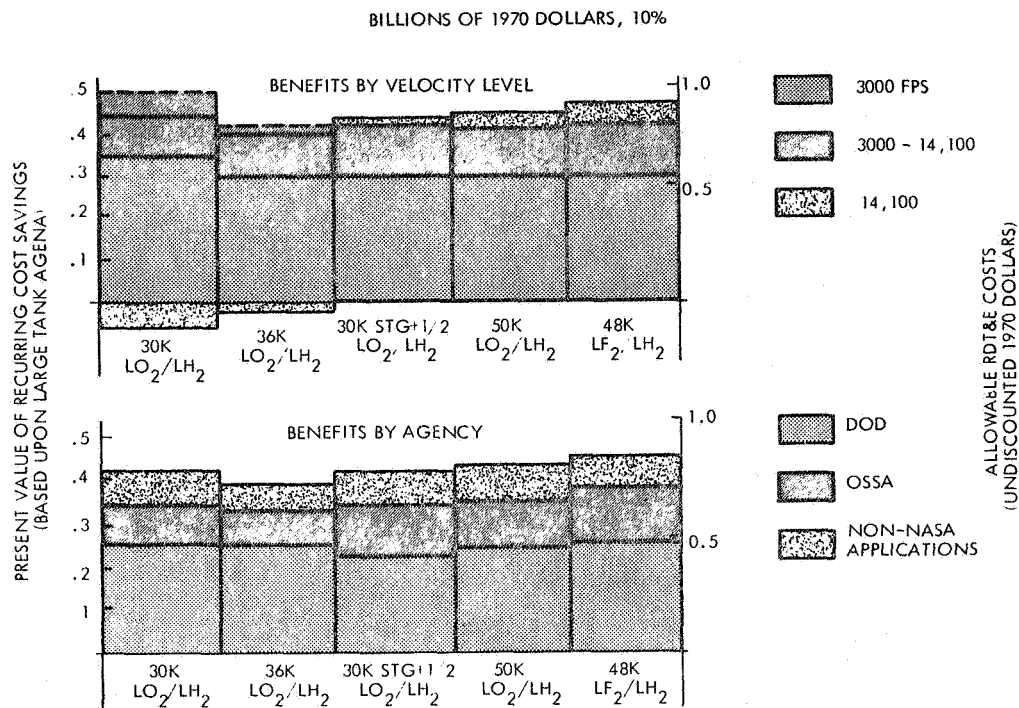
Figure 7-10 Benefits for Earth Observations Programs

Conclusions of the Economic Analysis

With consideration given to the analysis performed over Phases I and II, the following are the final economic conclusions:

1. A reusable Space Tug is a cost-effective investment evaluated at the 10 percent discount rate.

2. When consideration is given to the sensitivity analyses performed in this study, a 48,000 lb LF_2/LH_2 Tug appears to be the best (most cost-effective) investment. If on the other hand there are technological or other factors that call for selection of a LO_2/LH_2 propellant tug, the 50,000 lb size is the best investment.

3. Based upon the Large Tank Agena as a reference case, up to $1.75 billion could be spent for Space Tug development for it to be cost effective when evaluated at 10 percent discount rate. Furthermore, the development costs are justified by payload reuse benefits in orbits less than synchronous equatorial.

4. The Space Tug could return more than $1 billion in additional allowable non-recurring costs to the Space Shuttle/Tug system.

5. Given the groundrules of this study, a concurrent IOC of a reusable Tug with the Space Shuttle is desirable. However, conditions such as the Space Shuttle phase-in and NASA budget constraints could alter these results.

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Annex A
THE ECONOMIC PRINCIPLES UNDERLYING A COST-EFFECTIVENESS AND COST-BENEFIT ANALYSIS OF THE SPACE TUG

In the economic literature the terms cost-benefit and cost-effectiveness are sometimes used as equivalent terms. Cost-effectiveness analysis, in a strict sense, is only concerned with identifying technically feasible Space Tug systems that assure either the maximum of transportation capability at any given budget level or the minimum cost for any given capability requirement. Although in theory this is a rather straightforward task, in practice it may be very difficult to determine the cost-effective systems. Cost-benefit analysis in this case is the broader task of selecting a single Space Tug system from all of the possible cost-effective Space Tug candidates.

COST-EFFECTIVENESS ANALYSIS

In Figure A-1 the shaded area shows a hypothetical example of a cost-efficiency frontier, removed from the specific analysis of Space Tug systems. The horizontal axis represents expenditures or budget; the vertical axis represents an abstract univalue measure of capability. All points on the solid line represent the maximum capability, for a given technology, obtainable for a given expenditure (budget). These are the set of economically efficient points. Any point lying within the shaded area — the feasible region of project/cost combinations — is inefficient, i.e., by moving toward the left and upwards to the boundary line we improve the economics of systems choice. Cost-effectiveness analysis is concerned with finding projects where no increased capability (e.g. an increased amount of payload weight in synchronous equatorial orbit) is possible without a corresponding increase in budgetary outlays. Equivalently for systems that are cost effective, a decrease in cost is not possible without a corresponding decrease in capability. The set of cost-efficient points, the cost curve, is shown by the boundary of the shaded area, F_0, in Figure A-1.

By inspection we see that P_0, a point not on the frontier, is not cost effective. The system P_0 requires a budget of b_0 and promises a capability of k_0. We can find other
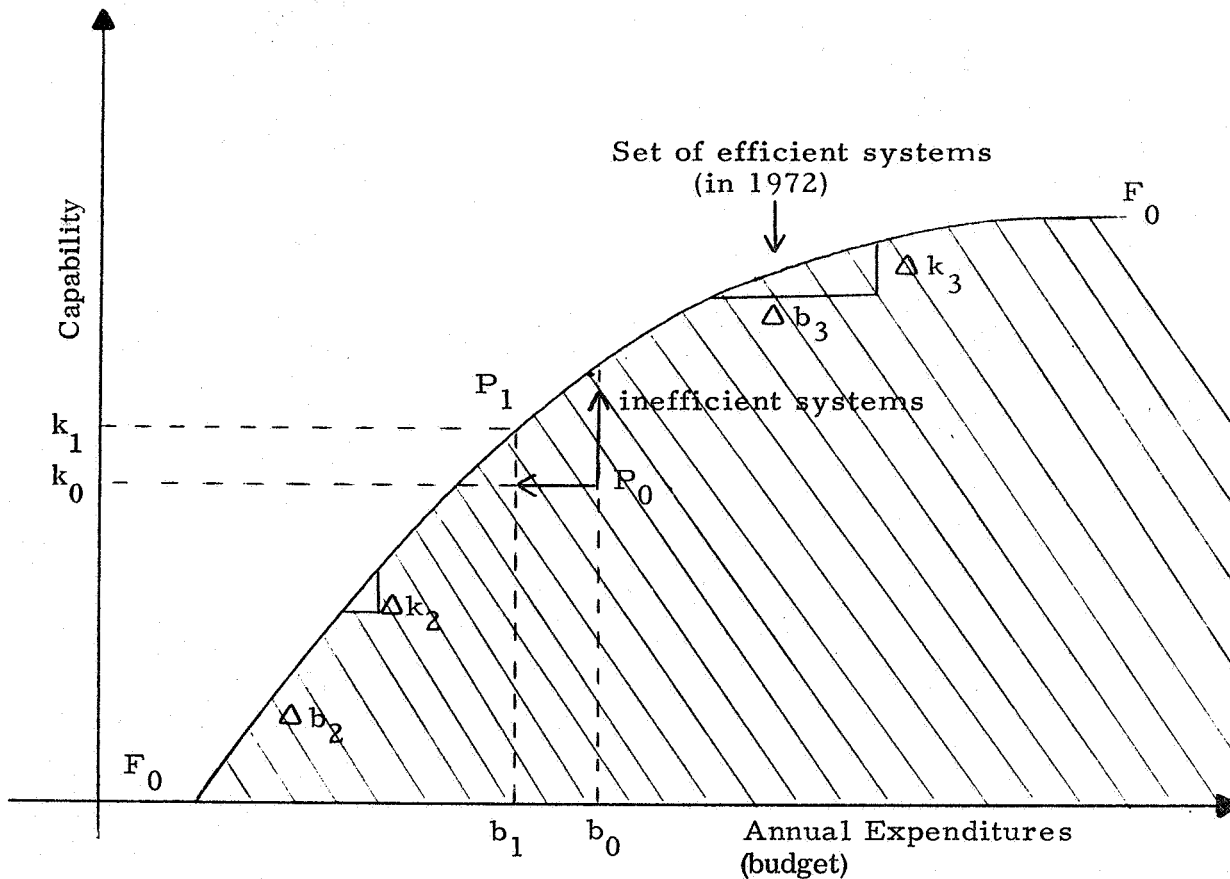
Figure A-1 The Cost-Efficiency Frontier

projects (plans), different from P_0, that offer more capability or less cost or both.
Such a system is shown at P_1, with a budget requirement of b_1 ($< b_0$) and a capability
of k_1 ($< k_0$).

From the shape of the cost-efficiency frontier we also observe that by increasing the
budget we add capabilities along the cost curve. As we move out to larger and larger
funding levels, any additional funding yields smaller and smaller increments in capa-
bility. In other words, the shape of the efficiency frontier reflects increasing marginal
costs as the capability requirements expand. The change in capability of Δk_2 is equal
to the change in capability of Δk_3 at a higher funding level. But the same absolute
increase in capability is bought at an increased incremental cost ($\Delta b_2 < \Delta b_3$).

In summary, cost-effectiveness analysis tries to answer two questions: first, what is the minimum cost for a given capability requirement k_0 (see Figure A-2) and, second, what is the maximum capability given a budget or funding level of b_0 (see Figure A-3). The first of these two approaches is commonly known as the equal capability approach, the latter is known as the equal budget approach. The set of answers to either of these questions will be a point on the efficiency frontier F_0. In this abstract example, the evaluation of different projects is based upon a single measurement of capability. As the capability requirements change, the marginal costs of added capabilities change. This is reflected in the shape of the cost-efficiency frontier.
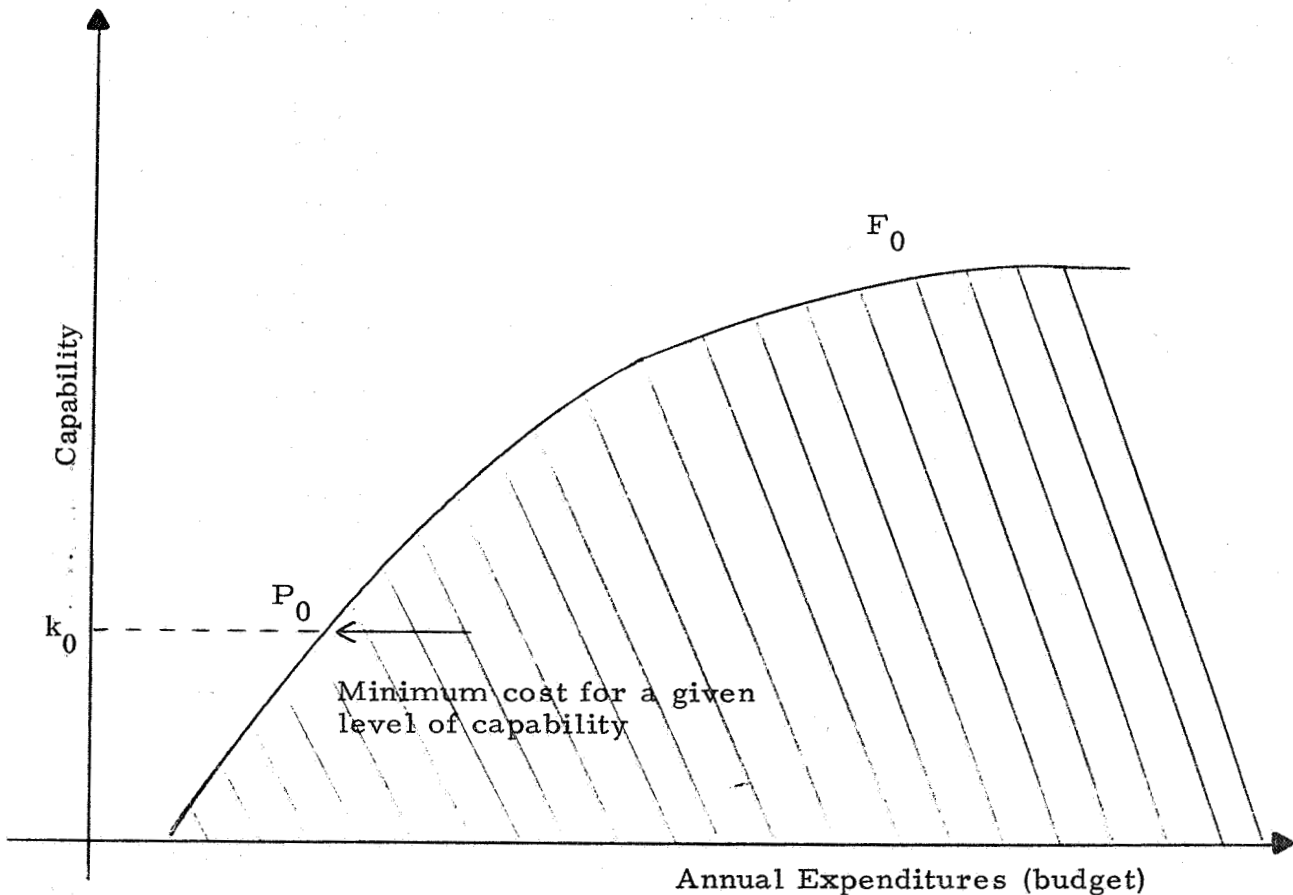
Figure A-2 Equal Capability Efficiency

A-3
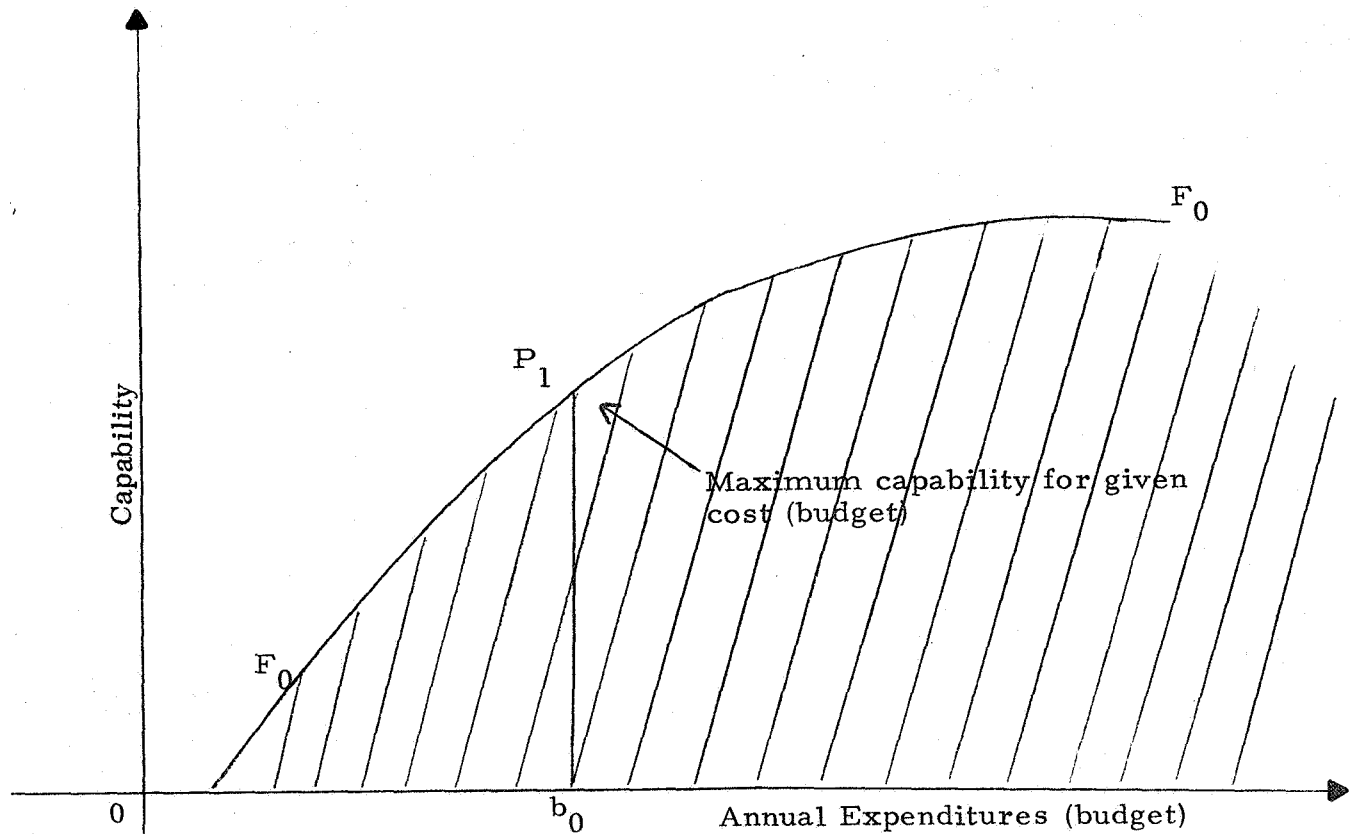
Figure A-3 Equal Budget Efficiency

Cost Effectiveness: Single Capability with Technological Change

A technological development such as a reusable Space Tug system will change the
efficiency frontier for space transportation systems and spacecraft programs. In
general, technological change will shift the efficiency frontier, F_0, of Figure A-1
upward and towards the left as shown in Figure A-4. If an efficient project is evalu-
ated prior to the introduction of the new technology, e.g., point P_0, it is immediately
apparent that P_0 is no longer cost effective with regard to the new efficiency frontier
F_1. That is, after technological change and innovation have taken place, other systems
can be found with the new technology, that provide the same capability at less cost
(P_1) or more capability at the same budget level (P_2).

A-4

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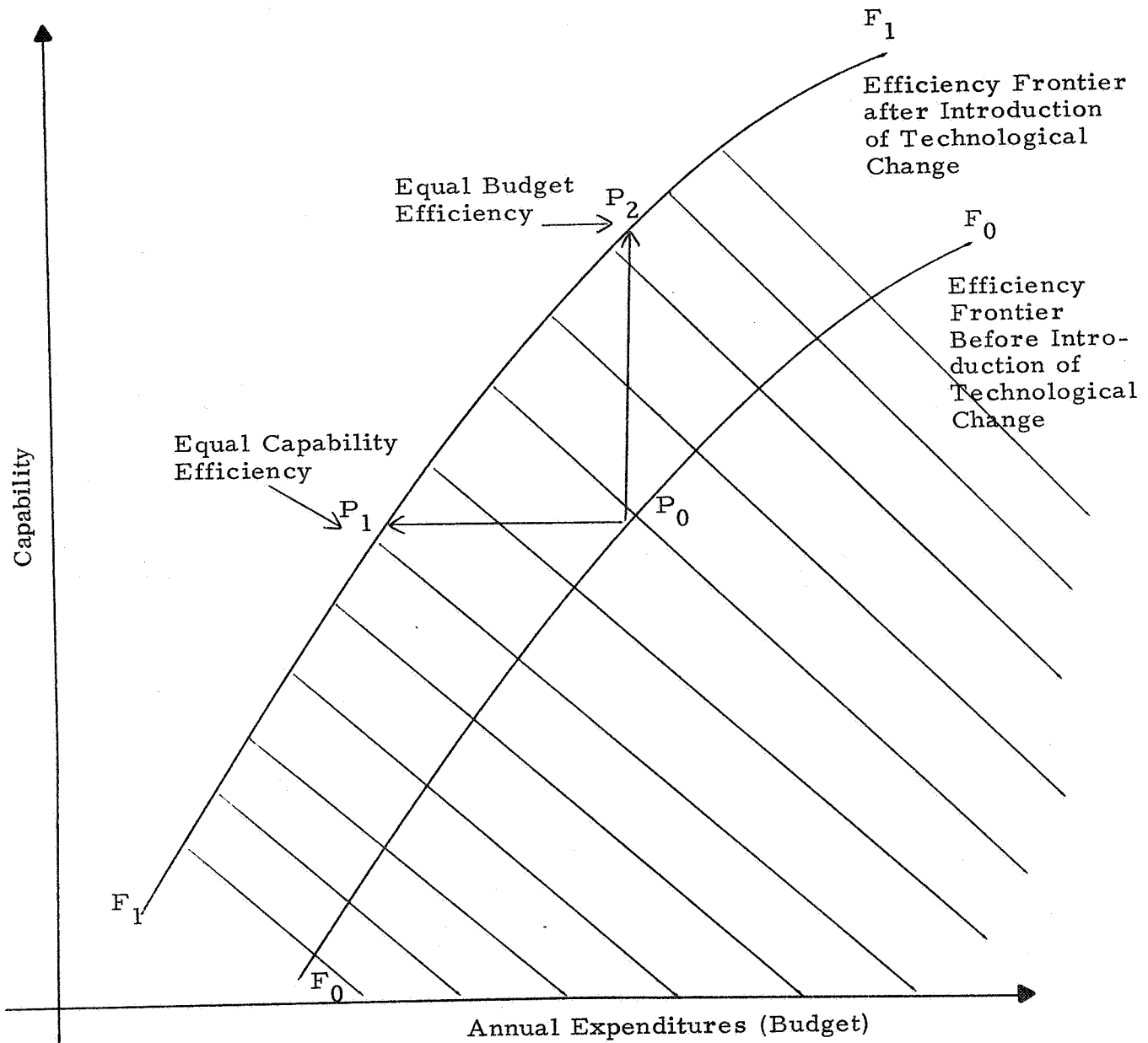
Figure A-4 A Shift in the Efficiency Frontier Caused by the
Introduction of Technological Change

Technological change does not always rain onto society in a steady stream; the more recent history of technology, especially in space related activities, suggests that technological change must commonly be purchased by (often substantial) investments in RDT&E and initial investment in new hardware. Suppose now that it is known with a fair degree of certainty that a given RDT&E effort will be capable of shifting the cost-effectiveness frontier from its present position (e.g., from line F_0 to line F_1 in Figure A-4). Within the confines of cost-effectiveness analysis (strictly defined), the following two questions may now be asked:

(a) Equal capability efficiency for a given capability level. What is the net cost saving that can be achieved by adopting the new technology, and are these cost savings (i.e., $P_0 - P_1$) large enough to justify the incremental non-recurring outlay on RDT&E and new hardware,[1] over the uselife of the new system?

and

(b) Equal budget efficiency. What increases in capability are brought about by technological change, at the same budget level, after the new system has been introduced, and will the economic value of this added capability justify the required, incremental outlays on RDT&E and new hardware over the uselife of the new system?[2]

Question (a) above is by far the easier one to answer from an empirical point of view and is the general approach of this analysis. In answering that question, one need only make the assumption that the expenditure on a capability prior to the development of the new technology represents the economic value of this capability to society. Based upon this assumption, a very conservative and objective estimate of the benefits from the new technology is the annual cost savings achievable at the activity level purchased under the old technology. If it is found that the total cost saving, aggregated over the uselife of the project and adjusted for the time value of economic resources,[3] more than covers the initial outlays on RDT&E and hardware for the new system, then one may unambiguously conclude that based upon cost-effectiveness the new system should be developed and adopted.

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[1] Excluding those costs that have been sunk.
[2] The reader will notice that these two questions are merely extensions of the equal capability and the equal budget approaches defined above.
[3] More about the time streams of costs and benefits will be discussed below.

It is much more difficult, in practice, to answer question (b) above. For the question really amounts to asking:

(b') Given the fact that we can increase our capability by the introduction of a new technology, does the economic value of the added capability justify the required additional expenditures up to an equal budget outlay?

Clearly, this question cannot be answered unless one can, in fact, place a value on the additional capability. In other words, question (b) really requires one to know demand curve of society for the activity in question.

Once we are involved in the placing of economic values, a further extension of cost-effectiveness analysis is suggested, i.e., cost-benefit analysis. In principle, at least, there is no reason why question (b') above should be confined to a unique budget outlay. One might just as legitimately ask whether the economic value of any additional capability justifies an expansion of the budget required to achieve it. That is, any addition to expenditures (budget) may be justified as long as the economic benefits associated with the incremental capability at least offset the incremental expenditure.

It is obvious, then, that cost-effectiveness analysis in the narrow sense of that term as defined above has at least one severe shortcoming: the approach abstracts entirely from the pertinent question whether or not marginal changes in project scale (i.e., in the proposed budget level or in the proposed effectiveness level) are economically desirable.[1] A fundamental theme of our argument is therefore that cost-effectiveness alone — either question (a) or (b) — constitutes an overly simplified view of the problem. Charles Hitch, writing in 1960 and again in 1965 on defense budget analysis as a whole, also urged that attention not be focused exclusively on the minimum cost and maximum capacity problems.[2] Instead, he suggested, one should be deliberate and choose budget levels and capability levels as long as a change appears to gain more than it costs. This, for federal agencies, is a difficult undertaking. Yet, to analyze fully a future system it should be addressed to the extent possible.

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[1] It is well here to assume that the decision maker's budget is flexible. After all, our purpose at this point is to stress overall principles.

[2] Charles Hitch and Roland McKean, The Economics of Defense in the Nuclear Age, Cambridge: Harvard University Press, 1960, and Charles Hitch, Decision Making for Defense, Berkeley, University of California Press, 1965.

COST-BENEFIT ANALYSIS: SINGLE CAPABILITY WITH BENEFIT MEASUREMENT

To do the wider task of cost-benefit analysis is to select a single system from all possible cost-effective candidates. For this, it is necessary to use a benefit (utility or value) measure of the various capability levels associated with the possible systems.

This choice process can be illustrated with the aid of Figure A-5 which shows the cost curve and the benefit curve confronting the decision-maker. It should be noted, first of all, that the cost curve in Figure A-5 differs substantially from that shown in Figure A-4. The latter denotes recurring costs per year as a function of capability per year. The cost curve in Figure A-4 on the other hand, refers to total program costs over the entire planning horizon. Because total program costs are incurred over time, it must be assumed that all costs are adjusted for the time value of economic resources by means of discounting future costs. The time stream of program benefits, summed up in the benefit curve, also is assumed to have been discounted appropriately.

Figure A-5 illustrates the general relation between cost, capability and benefits. Observe that at higher and higher levels of capability, an additional (marginal) unit of capability becomes increasingly more costly (i.e., the marginal cost of capability increases) while, at the same time, the marginal benefit derived becomes increasingly smaller. The assumption of progressively decreasing marginal benefits is based on the notion that successive additions to the number of pounds of payload in orbit will perform successively less valuable tasks. Indeed there may well be a saturation point, which means that the benefit curve in Figure A-5 will eventually become vertical.

At a given level of capability, say k_0, net program benefit is measured by the horizontal distance between the benefit and the cost curves. In Figure A-5 the net benefit at k_0 is given by the distance CD; at k_1, it is given by AB. The cost curve is really an efficiency frontier that associates a given level of capability with the least cost system which will provide that capability. The optimal system is therefore the one
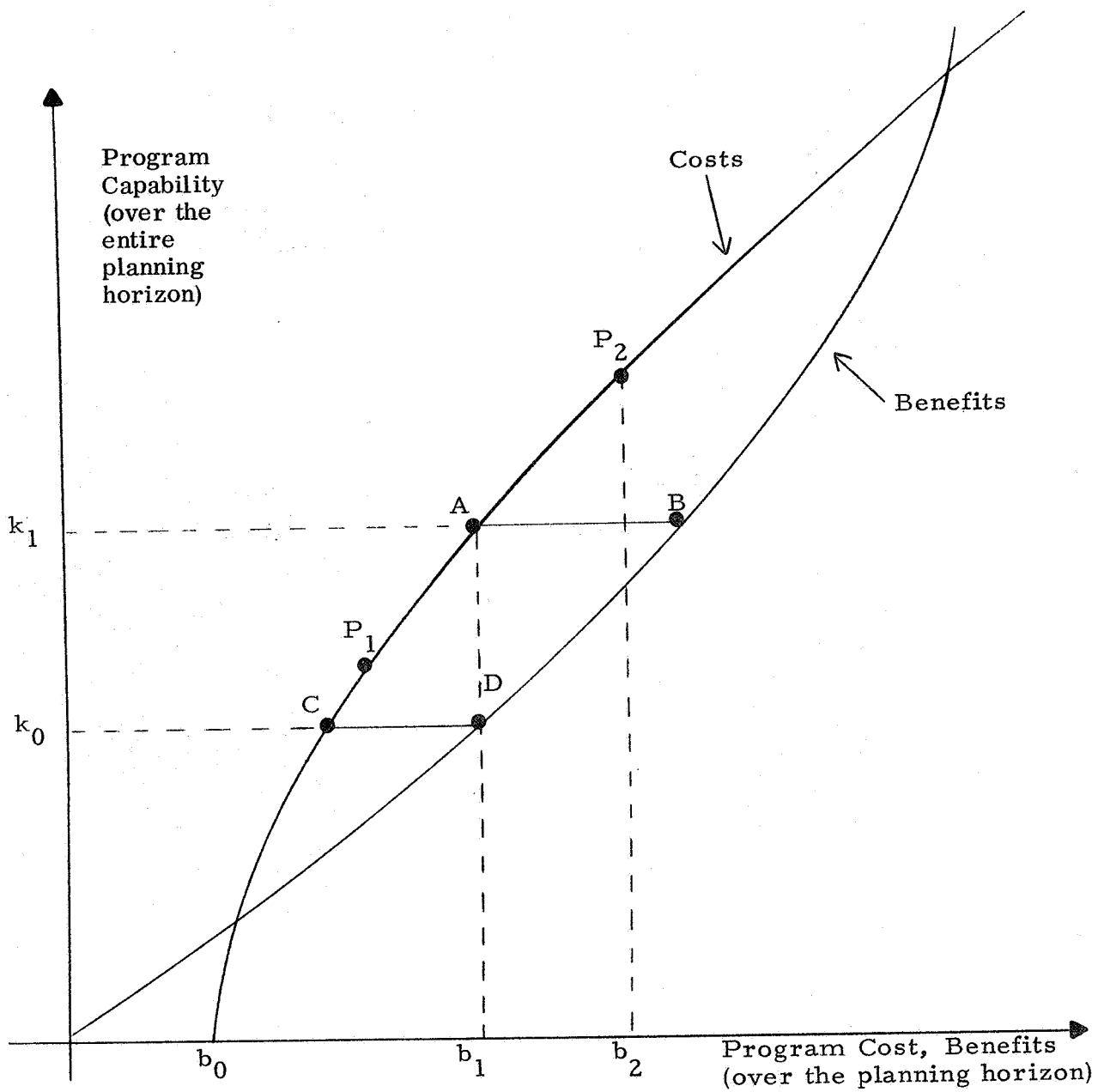
Figure A-5 Cost-Benefit Analysis

corresponding to the capability level at which the distance between the benefit and the cost curves, i.e., the net benefit, is maximized. It is the capability level at which the cost curve and the benefit curves have the same slope, i.e., at which marginal benefits are just equal to marginal costs. In Figure A-5 this optimum capability level is k_1.

Some cost analysts prefer to define the optimum capability level (and the corresponding optimum system) as that level at which the ratio of program benefits to program costs is maximized. In Figure A-5 that level might be capability k_0 at which, however, the net program benefit is seen to be suboptimal.[1] The so-called benefit/cost ratio therefore is not a reliable criterion of program evaluation while the net-program-benefit criterion generally leads to the economically correct choices.

Having established these fundamental points, suppose a new system is introduced. We now observe that shifts in the efficiency frontier (cost curve) can occur at different rates as the capability increases.

If the shift results in larger cost savings at higher levels of capability, it is quite possible that the capability level should be increased even beyond the original cost of the old system — and that one can do so with economic gain. Figure A-6 illustrates the case where the benefit-cost choice lies between the equal capability and equal cost points, and Figure A-7 the case of increased cost and increased capability.

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1 These two criteria will not usually lead to the same choice. Let B(Q) denote the benefit function, C(Q) be the cost function and B'(Q) and C'(Q) be the first derivatives, with respect to Q, of these two functions, respectively. Let Q denote capability levels.

Then net benefits are maximized at that level Q at which B'(Q) = C'(Q), i.e., where marginal benefits equal marginal costs.

The cost benefit ratio, on the other hand, is maximized (subject to second order conditions) at a level Q such that $\dfrac{\partial B/C}{\partial Q} = 0$, which implies

$$\frac{B'(Q)}{B(Q)} = \frac{C'(Q)}{C(Q)}$$

It is clear that, in general, these two first-order conditions are not satisfied at the same level of Q, i.e., the net benefit and benefit/cost ratio will not lead one to choose the same capability-budget point.
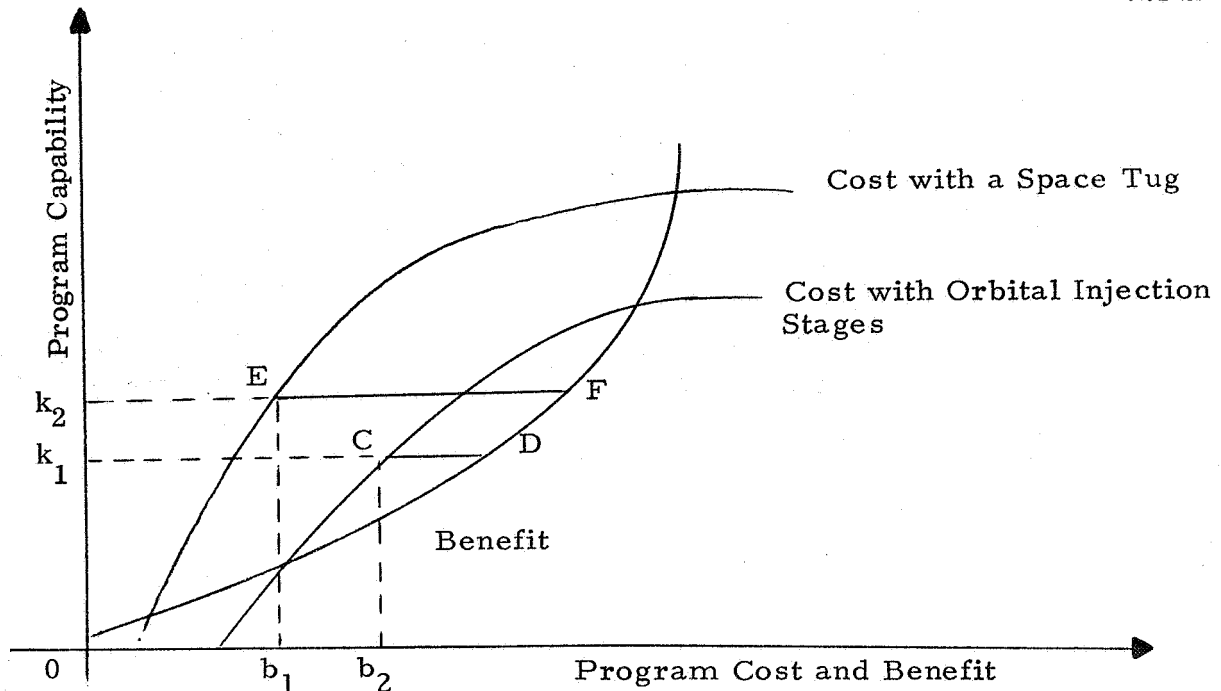
Figure A-6 Effect of Technological Change on Optimum Investment:
Reduced Costs and Increased Capability

In Figure A-6, the maximum difference between benefit and costs for the old system is given by the line segment CD , while that for the new system is given by the line segment EF . The optimal level of capability therefore increases from k_1 to k_2 , and the optimal budget falls from b_2 to b_1 . However, this budget decrease is a result of the particular shape of the curves drawn in Figure A-6. Figure A-7 illustrates a case in which the optimal budget actually increases as a result of the introduction of the new (Space Tug) system. The increase in the optimal capacity causes a rise in total budget that more than offsets what would have been the decrease in cost associated with the same level of capability.

Thus, a new cost curve involving a downward shift in relative costs may nevertheless yield an optimal level of capability involving a greater budget than before. (See Figure A-7.) The condition for this is easily described: at the current level of total expenditures the marginal increase in benefits must more than offset the marginal increase in the new costs.

Even this rather simple model strongly suggests that the benefit associated with the increased capacity provided by a Space Tug system is a rather important component of the system selection decision process. Mere cost minimization or capacity maximization will not suffice.

A-11
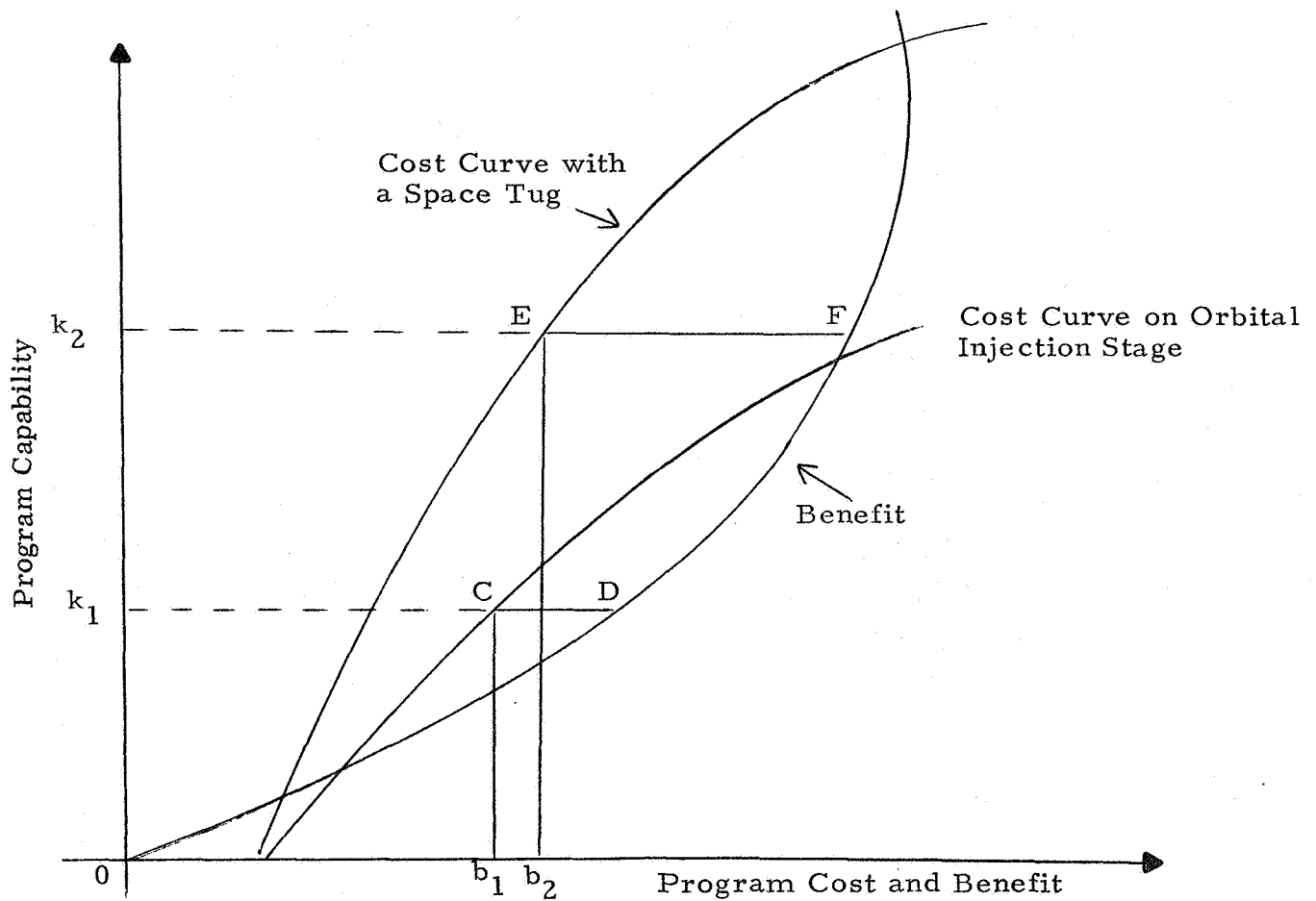
**Figure A-7 Effect of Technological Change on Optimum Investment:
Increased Capability and Increased Costs**

In actual practice, one may, of course, have to assume that an agency's budget is relatively fixed and will be spent.   In such situations, government agencies are restricted in their practical analysis of projects, to questions (a) and (b) before, or, perhaps to a choice of systems somewhere in between the segment defined by P_1 and P_2 on F_1 in Figure A-4.   If a program can be justified in terms of either cost-savings at P_1 or capability increases at P_2, then the program is certainly based on firm grounds.

CRITICAL PARAMETERS IN THE EVALUATION OF A PUBLIC EXPENDITURE

On the definition of the benefits and costs of a Space Tug system, we shall now turn to an examination of the major parameters entering the economic and social evaluation of the system, namely: the social rate of discount, the investment horizon, and costing for economic analysis.

The Social Rate of Discount

The Theoretical Underpinnings of the Concept. Briefly, society's rate of time preference may be defined as a rate of interest which reflects the consumers subjective, relative evaluation of given quantities of consumables available at different points in time. For example, if, in year 0 , consumers assign the same value to 100 units of consumables available immediately as they do to the certain prospect of receiving 105 units of consumables one year hence, then their rate of time preference is said to be $0.05 = \left[\dfrac{105}{100} - 1 \right]$ = 5 percent. Alternatively, the rate of time preference may be defined as the rate of interest which consumers would have to be offered in order to persuade them to sacrifice additional current consumption in favor of additional future consumption.

Any investment project — public or private — involves the sacrifice of consumables at some point in time for the sake of increased consumption at one or more subsequent points in time. From the very definition of the rate of time preference, it is clear that this rate must somehow be reflected in the social rate of discount used in the evaluation of public projects.

There is, however, still another side to the social discount rate: the social opportunity costs of a public project are the benefits foregone when the economic resources used by the project are diverted from the private to the public sector. The social rate of discount should reflect these opportunity costs as well.

Let us assume, for example, that all of the resources devoted to a public project would have been used in the private sector for investment outlays promising an annual

rate of return of 10 percent before corporate income taxes and after an allowance for the eventual replacement of worn out equipment. Suppose $1 billion in resources were transferred to the public project. Then the public project could be justified economically only if it also promised a benefit stream (necessarily accruing to members of the private sector at large) equivalent to an annual benefit stream of $100 million (10 percent of $1 billion). An alternative way of expressing this is that the present value of the benefit stream produced by the public project, discounted at $r = 10$ percent, must be at least as high as $1 billion, or, that the net present value (NPV) of the project must be greater than or at least equal to zero.

The interest rate concept used in the preceding paragraph is sometimes referred to as the time productivity of economic resources. It is the rate of return that society is able to earn in the private sector by sacrificing current consumption in favor of future consumption, i.e., by investing economic resources in productive investment projects. In contrast, society's rate of time preference is the rate of return for which society is willing to sacrifice current consumption for the sake of increased future consumption. These two interest-rate concepts should not be confused: the rate of time productivity is an objective, technical concept; the rate of time preference, on the other hand, is a purely subjective magnitude.

It can be shown that, in the imaginary world of classical economics, the savings and investment behavior of society — the nation's capital markets — would always drive the economy to an equilibrium position in which all individuals exhibit the same (social) rate of time preference, all investors face the same (social) rate of time productivity and in which, moreover, the social rate of time preference would be just equal to the social rate of time productivity. This overall equilibrium market rate of interest would then be the appropriate discount rate to be used for public-project evaluation.[1]

Unfortunately, the real world differs significantly from the happy state of affairs in the classical model. For one, individual investors face different degrees of risk and differ

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[1] See MATHEMATICA, Inc., "On the Principles of Public Project Evaluation," in Cost Benefit Analysis of New Launch Systems, Vol. 1, July, 1970.

in their attitudes toward risk. The rate of returns required by private investors there-
fore include risk premiums which differ over the spectrum of investors.

Secondly, the tax system does not treat all investors in the private sector equally.
Corporations, for example, force tax rates that differ from those paid by unincorporated
businesses, and there are also differences in the rates paid by different unincorporated
business firms. To earn the same after-tax rate of return, different business firms
must therefore earn different pre-tax rates of returns on their marginal investments.

Finally, net-savers in our economy typically obtain rates of return on their savings that
differ from the rates faced by net borrowers. Different consumers therefore are char-
acterized by different rates of time preference.

In short, then, in the real world there exists no single market rate of interest which
can be viewed as the appropriate discount rate for public project evaluation. The rate
being used for that purpose must therefore be a weighted average of the various rates
prevailing in the market.

In the real world, a resource transfer from the private to the public sector does not
usually come solely from private investment projects: part of the resources will surely
come from private consumption. It follows that the opportunity costs of the resource
transfer must reflect not only the spectrum of rates of return on foregone private in-
vestments, but also the spectrum of time preference rates of those who sacrificed
current consumption. This requirement confronts one with enormous difficulties in
any attempt to estimate the appropriate level of the social discount rate for practical
applications of benefit-cost analyses.[1] Suffice it to say that the fundamental idea under-
lying this estimation process is always the same: one seeks to estimate the magnitude
of the sacrifice borne by the private sector when resources are transferred from pri-
vate consumption or investment to public-sector use, and to express this sacrifice in
the form of an annual rate of return, r .

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[1]This has been thoroughly dealt with in <u>Ibid.</u>

THE DETERMINANTS OF THE EVALUATION HORIZON OF A PROJECT

The assumed economic uselife of an investment project is normally something shorter than infinite because of one or a combination of the following factors.

1. Factors inherent in the project itself

   a. One of the physical inputs to the project depreciates over a period of time, collapses at a point in time (one-hoss-shay depreciation) or becomes unavailable at a point in time (e.g., a rented piece of land, or an exhaustible supply of raw materials).

   b. The demand for the product or service yielded by the project may drop off or disappear altogether after some time.

2. Factors inherent in the decision-maker

   a. The decision-maker is risk-averse and deliberately chooses a finite and possibly short investment horizon as a risk adjustment.

   b. The decision-maker limits the investment horizon to his own life expectancy.

Since the present discussion is concerned exclusively with public investments in transportation systems, item 2(b) above can be dismissed from consideration altogether. Furthermore, it has been argued in the earlier report by Mathematica cited previously and in the pertinent economic literature at large[1] — that the government should not be risk-averse in evaluating alternative public projects. This means that a public agency should not, because of being risk-averse, shorten the investment horizon (N) of a public project arbitrarily. On the basis of this argument, item 2(a) above can be eliminated from consideration as well.

With respect to item 1(b) above, it can probably be assumed that with growing industrialization and population density there will continue to be a steady — or even increasing — demand for earth observation, at least for the next four to five decades. But at discount rates greater than, say, 5 percent, the present value of a steady stream of annual benefits increases only at a sharply diminishing rate with increases in the investment horizon, as is indicated in Figure 8.

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[1] See, for example, K. J. Arrow and B. C. Lind.
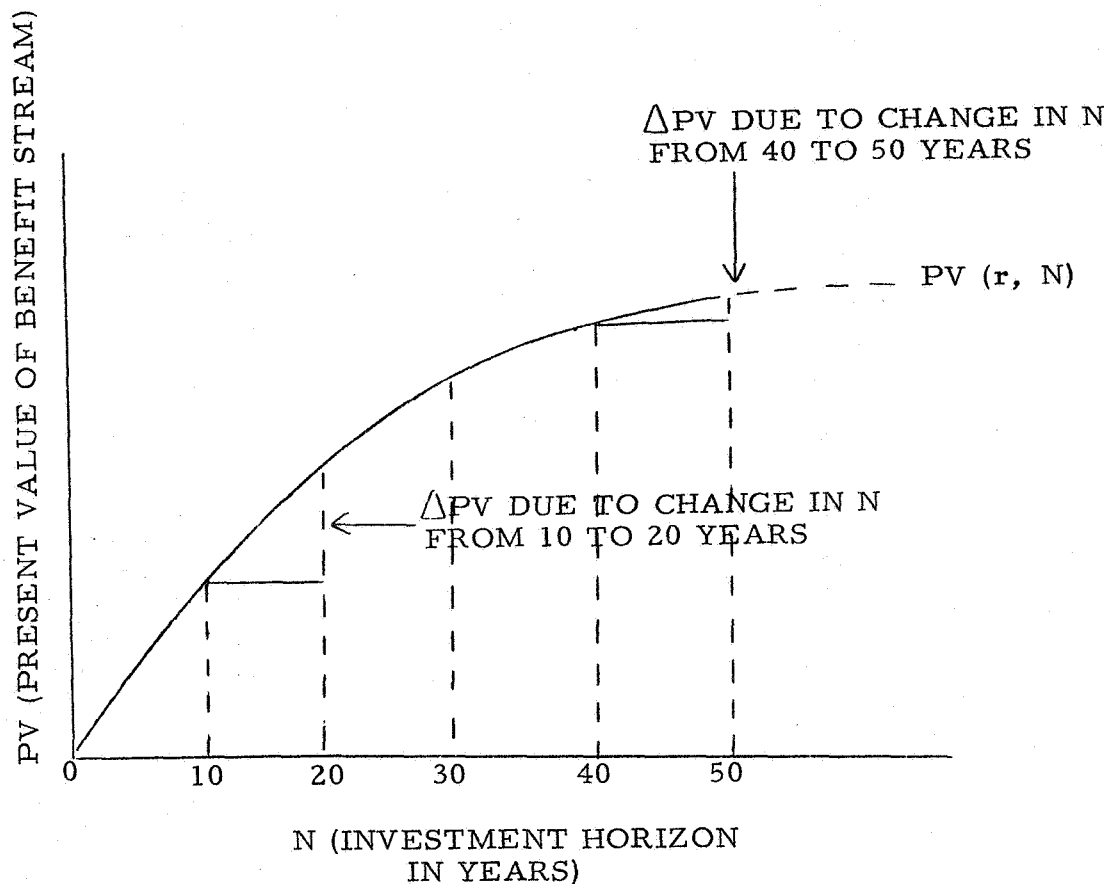
Figure A-8 The Effect of the Investment Horizon (N) on the Present Value
of a Steady Stream of Benefits

In Figure A-8, the symbol PV ($\bar{r}$, N) denotes the present value of a steady stream of
annual benefits obtained for N consecutive years and discounted at some discount
rate $\bar{r}$ > 5 percent. As may be inferred from Figure A-8 the assumption of a 40 or 50
year project horizon is almost tantamount to assuming, for purposes of evaluation, an
infinite horizon. Thus, if it is reasonable to assert that the demand for earth-oriented
remote-sensing programs will continue into the indefinite future, one really needs to be
certain only that it will continue for at least the next four to five decades.

This leaves us with point 1 (a) above, i. e., with the question or whether a physical
input into a Space Tug System will become unavailable at some future point in time,
and if so, when.

A-17

Since the blueprints and documentation for a Space Tug system exist and any number of identical or upgraded spacecraft can be built, point 1 (a) can also be eliminated from consideration.

The argument for an infinite horizon evaluation may be made from a second viewpoint which is made with reference to Figure A-9.
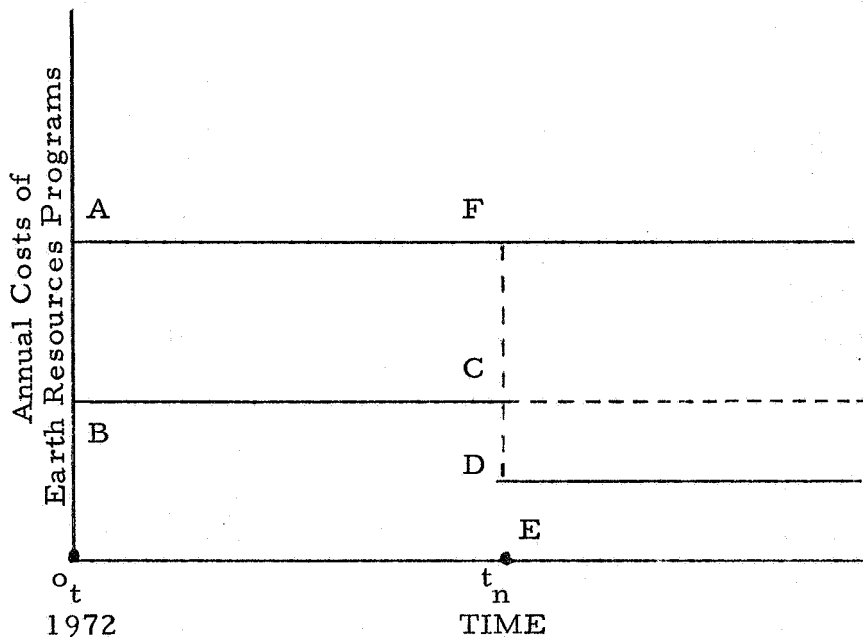
Figure A-9 Illustration of the Project Horizon

The assumption is made, as above, that the U.S. space program will continue into the indefinite future. It is further assumed that the annual cost of the U.S. space program by conventional means is OA and with the aid of a Space Tug technology it is OB. The economic benefits attributable to the Space Tug technology are BA per year.

Based upon an equal capability analysis, it is expected that at some point in time, say, t_n , that a technological advance will occur that further reduces the cost of the program to ED per year, realizing an additional savings of DC per year. It would be an error to attribute to the new technology a value of DF in annual savings even though it replaces the Space Tug technology that will under study. Any decision to introduce the new

technology should be based upon its incremental benefits, DC versus its incremental developmental costs. So long as there is a U.S. space program, the original savings should be attributed to the Space Tug technology, which is an infinite horizon approach.

COSTING FOR ECONOMIC ANALYSIS

As a final element to this section, we draw attention to the issue of costing for economic analysis.

Basically, the idea is to include only those costs that are relevant to an analysis of the benefits and incremental costs of an ongoing unmanned space program. Therefore all sunk costs must be identified and removed. By sunk costs, we mean all expenditures already made or committed regardless of whether or not the U.S. unmanned space program is continued. These include the RDT&E and unit investment costs of existing spacecraft, launch vehicles, and stages, but exclude the cost of necessary RDT&E and additional spacecraft that are required if the program continues over the period that we are examining.

Once the sunk costs are culled, the following scheme, as depicted in Figure A-10 of cost organization is useful:
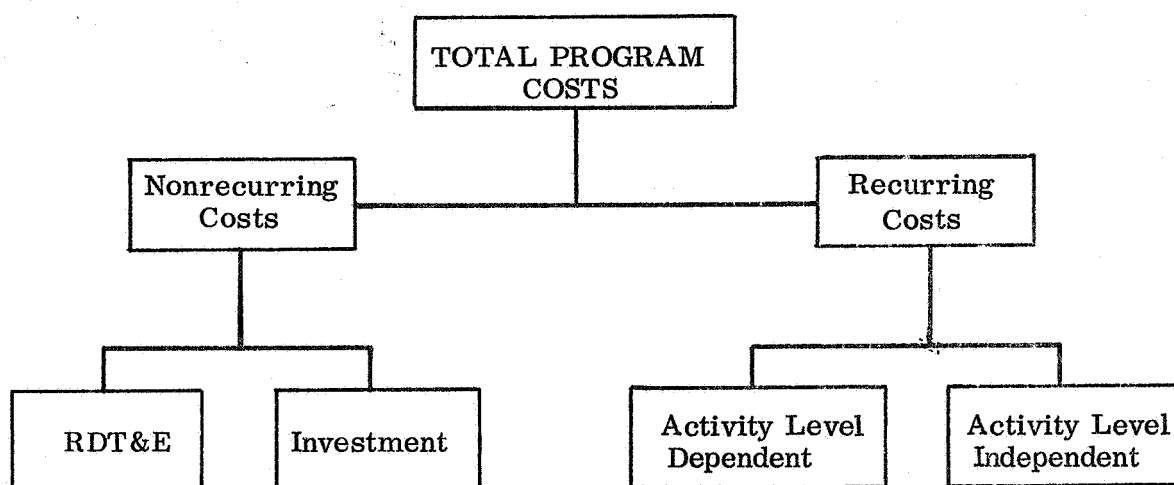
Figure A-10 An Organization of Program Costs for Economic Analysis

Nonrecurring costs consist of additional RDT&E and investment costs that are required for the implementation of the U.S. unmanned space program over the period under investigation.

It is important to separate the recurring costs that are activity level independent and activity level dependent (incremental) costs. This is because we will be concerned with uncertain states of demand for space activity and it will be important to ascertain the sensitivity of marginal cost to marginal output.

Annex B
MIXED INTEGER PROGRAMMING FORMULATION
OF THE SPACE TUG PROBLEM

An important issue in the Space Tug selection process is the composition and time phasing of the Tug fleet mix. Annex B describes the approach and principal results of an analysis performed by Mathematica to develop a tool that could resolve this question. The result of this analysis was a computer program, called OPCHOICE, which was developed and operated on a demonstration basis by Mathematica during the Space Tug Economic Analysis study.

COMPOSITION OF THE OPCHOICE MODEL

The OPCHOICE computer program approaches the Tug fleet mix/phasing problem using a mixed integer programming formulation that operates by minimizing total program costs. Total program cost is defined to be the sum of RDT&E expenses, investment, and recurring (i.e., direct) operations costs, diminished by any payload reuse benefits which may be generated. To express this mathematically, we define the following:

c_j = Undiscounted RDT&E costs for Tug of type j, for

$j = 1, 2, \ldots, J$

f_j = Undiscounted investment cost to make one Tug of

type j, $j = 1, 2, \ldots, J$

r_{ij} = Undiscounted recurring cost for Tug j to perform

mission i, $i = 1, 2, \ldots, I; j = 1, 2, \ldots, J$

ρ_{ij} = Undiscounted recurring cost for Tug j to retrieve

payload i when tug is flown in the "retrieve only"

mode $i = 1, 2, \ldots, I; j = 1, 2, \ldots, J$

g_i = Undiscounted benefit realized by reusing a payload of type i rather than making it again, $i = 1, 2, \ldots, I$

λ_j = Number of years required for RDT&E for Tug j, $j = 1, 2, \ldots, J$

υ_j = Number of years required to build a Tug of type j, $j = 1, 2, \ldots, J$

η_j = Number of years beyond date of availability that investment costs for Tug j will be spread $j = 1, 2, \ldots, J$

σ_i = The number of years after a payload is recovered that it can be reused, $i = 1, 2, \ldots, I$

a_{ij} = $\begin{cases} 1 \text{ if Tug } j \text{ can perform mission } i, \, i = 1, 2, \ldots, I; \\ \quad j = 1, 2, \ldots, J \\ 0 \text{ otherwise} \end{cases}$

α_{ij} = $\begin{cases} 1 \text{ if tug } j \text{ can retrieve payload } i \text{ when Tug is} \\ \quad \text{flown in the "retrieve only" mode } i = 1, 2, \ldots I; \\ \quad j = 1, 2, \ldots, J \\ 0 \text{ otherwise} \end{cases}$

$P_j(s)$ = Spreading function for RDT&E costs for Tug j, $j = 1, 2, \ldots, J$

$Q_j(s)$ = Spreading function for investment costs for Tug j, $j = 1, 2, \ldots, J$

v = $(1 + d)^{-1}$, where d = social discount rate

T = Total number of years in the mission profile

z_{ijt} = The number of flights in year t that Tug j will perform to retrieve payload i when Tug is flown in the "retrieve only" mode $i = 1, 2, \ldots, I$; $j = 1, 2, \ldots, J$; $t = \Lambda + \omega_i + 1, \ldots, T - \sigma_i$

T_o = Total number of years during which Tug could be developed (i.e., after a given date, we assume that no more Tugs can be made).

$$\delta_{jt} = \begin{cases} 1 & \text{if Tug } j \text{ is to be developed and available for the first time in year } t, \ j = 1, 2, \ldots, J; \ t = \lambda_j + \lambda_j + 2, \ldots, T_o \\ 0 & \text{otherwise} \end{cases}$$

$$\gamma_{jt} = \begin{cases} \epsilon_j & \text{where } \epsilon_j \text{ is the number of Tugs of type } j, \text{ if they are available for the first time in year } t \\ & j = 1, 2, \ldots, J; \ t = \lambda_j + 1, \ \lambda_j + 2, \ldots, T_o \\ 0 & \text{otherwise} \end{cases}$$

x_{ijt} = The number of flights in year t flown by Tug j in order to perform mission i, $i = 1, 2, \ldots, I$; $j = 1, 2, \ldots, J$; $t = \lambda_j + 1, \ \lambda_j + 2, \ldots, T$

y_{it} = The number of payloads of type i recovered in year t, $i = 1, 2, \ldots, I$; $t = \Lambda + \omega_i + 1, \ldots, T - \sigma_i$

w_{it} = Number of retrieved payloads of type i that are sent up again in year t, $i = 1, 2, \ldots, I$;

$$t = \Lambda + \omega_i + \sigma_i + 1, \ldots, T$$

$$\Lambda = \min_{j} \lambda_{j}$$

$\omega_{i} = $ Number of years that a payload of type i must remain in orbit, $i = 1, 2, \ldots, I$

RDT&E Costs

We can now express the RDT&E costs that are incurred as

$$\sum_{j=1}^{J} \sum_{t=\lambda_{j}+1}^{T_{o}} c_{j} \left[\sum_{s=1}^{\lambda_{j}} P_{j}(s) \, v^{t-\lambda_{j}+s-2} \right] \delta_{jt}$$

Note that the summation over time need only go from $t = \lambda_{j}+1$ for each j since it takes λ_{j} years to do all RDT&E for Tug j. Hence, by definition of δ_{jt}, $\delta_{jt} = 0$ for all $t < \lambda_{j}$. Furthermore, this summation need not go beyond the T_{o}-th year since no new Tugs may be developed after year T_{o}. Hence, $\delta_{jt} = 0$ for $T_{o} < t \leq T$.

Investment Costs

Investment costs can be written as

$$\sum_{j=1}^{J} \sum_{t=\lambda_{j}+1}^{T_{o}} f_{j} \left[\sum_{s=1}^{\upsilon_{j}+\eta_{j}} Q_{j}(s) \, v^{t-\upsilon_{j}+s-2} \right] \gamma_{jt}$$

where the same comments apply with regard to the summation over time. However, note that the upper limit of the summation is $\upsilon_{j}+\eta_{j}$.

The effect of summing over η_j more years is to allow investment costs to be spread an additional η_j years beyond the date that Tug j first becomes operational.

Recurring Operations Costs

Recurring costs of the first type are incurred each time a payload is sent up into orbit. These costs are

$$\sum_{i=1}^{I} \sum_{j=1}^{J} a_{ij} \, r_{ij} \sum_{t=\lambda_j+1}^{T} v^{t-1} \, x_{ijt}$$

Recurring costs of the second type are incurred when a Tug is sent up solely to retrieve a payload -- i.e., in the "retrieve only" mode. These costs are

$$\sum_{i=1}^{I} \sum_{j=1}^{J} \alpha_{ij} \, \rho_{ij} \sum_{t=\Lambda+\omega_i+1}^{T-\sigma_i} v^{t-1} \, z_{ijt}$$

Payload Reuse

Payload reuse benefits can be represented by

$$-\sum_{i=1}^{I} g_i \sum_{t=\Lambda+\omega_i+\sigma_i+1}^{T} v^{t-1} \, w_{it}$$

Various constraints are imposed on the values of the variables, γ_{jt}, δ_{jt}, ϵ_j, w_{it}, x_{ijt}, y_{it}, and z_{ijt}.

Mission Model

Let

D_{it} = Number of satellites of type i which must be put into orbit during year t.

The first constraint is that we satisfy the mission model. Mathematically, this is:

$$(1) \quad \sum_{j=1}^{J} a_{ij} x_{ijt} = D_{it} \qquad \text{for } i = 1, 2, \ldots, I \\ t = \wedge + 1, \wedge + 2, \ldots, T$$

Note that t starts at $\wedge + 1$ because this is the first year that any satellite could be orbited.

The remaining constraints are all definitional in that they are required to define a mathematically sound model. In other words, they are not external economic constraints. No title is given to these, but the meaning and implications of each are given.

If $x_{ijt} > 0$ or $z_{ijt} > 0$, we must impose a constraint to insure that the RDT & E costs are incurred for Tug j, and these costs must be incurred in the years ending just before the first year for which $x_{ijt} > 0$ or $z_{ijt} > 0$. Letting M be a large number, we impose this constraint with

(2) $\quad \sum\limits_{i=1}^{I} [a_{ij} x_{ijt} + \alpha_{ij} z_{ijt}] \leq M \sum\limits_{r=\lambda_j + 1}^{t_o} \delta_{jr}$

where $\quad t_o = \min(t, T_o)$

for $j = 1, 2, \ldots, J$ and $t = \lambda_j + 1, \lambda_j + 2, \ldots, T$

Note that if the left-hand side of the inequality is non-zero, the only way the inequality can be satisfied is for at least one of the δ_{jr} to be non-zero. Since the δ_{jr} are all to be declared as integer zero-one variables, the inequality would force at least one of the δ_{jr} to be 1. That only one of the δ_{jr} for the given j will be 1 follows from the fact that the objective function to be minimized is made smaller by having as few $\delta_{jr} = 1$ as possible. Having no other constraint which would possibly required more than one δ_{jr} to be 1 for the same j, insures the desired result.

Three sets of equations are necessary to insure that γ_{jt} is the number of Tugs of type j if they are available for the first time in year t, and is zero otherwise. The first of these three sets defines ϵ_j to be the number of Tugs of type j to be built, regardless of the year in which they are made available.

Let

N_j = The number of flights that can be made by a Tug of type j (also called the mean life)

Then a lower bound on ϵ_j is defined by the following:

(3) $\quad \displaystyle\sum_{i=1}^{I} \sum_{t=\lambda_j + 1}^{T} [a_{ij} x_{ijt} + \alpha_{ij} z_{ijt}] \le N_j \epsilon_j \quad$ for $j = 1, 2, \ldots, J$

That ϵ_j will not be any larger than this lower bound will be imposed by the next set of constraints.

To ensure that the cost for building these tugs is incurred in the proper years (i.e., if $\delta_{jt} = 1$, then the cost was incurrred in the ν_j years immediately preceding year t and in the η_j years following and including year t), we impose:

(4) $\quad \epsilon_j - \gamma_{jt} \le M (1 - \delta_{jt}) \qquad\qquad$ for $j = 1, 2, \ldots, J$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad t = \lambda_j + 1, \lambda_j + 2, \ldots, T_o$

If for some j and t, $\delta_{jt} = 1$, then the equation reduces to

$$\epsilon_j - \gamma_{jt} \le 0$$

or $\qquad\qquad\qquad\qquad \epsilon_j \le \gamma_{jt}$

The minimization of the objective function would tend to make γ_{jt} as small as possible. Thus, in this case, the value of γ_{jt} would be forced down to the value of ϵ_j. But ϵ_j is itself a variable and would therefore be forced down to the smallest value allowed by (3). Hence, we accomplish two things with this constraint: (a) force ϵ_j to take on the smallest allowable value as given by (3), and (b) force γ_{jt} to take on this value.

For this value of j, and for all $t' \ne t$, $\delta_{jt'} = 0$. The constraint would then be equivalent to

$$\epsilon_j - \gamma_{jt'} \leq M$$

or

$$\epsilon_j \leq \gamma_{jt'} + M$$

The value ϵ_j has already been properly constrained since there was a t for which $\delta_{jt} = 1$. Hence, we are only concerned with the value of $\gamma_{jt'}$. Again, minimization of the objective function will insure that $\gamma_{jt'}$ is as small as possible, or in other words, zero.

If there exists a j for which $\delta_{jt} = 0$ for all t (i.e., this Tug is not to be developed) then for all t, (4) says:

$$\epsilon_j - \gamma_{jt} \leq M$$

or $\qquad \epsilon_j \leq \gamma_{jt} + M$

In this case, the minimization of the objective function would ensure that $\gamma_{jt} = 0$ for all t, but ϵ_j would not be constrained to zero as it must. To guarantee this condition, we add the constraint.

(5) $\quad \varepsilon_j \leq M \sum\limits_{t=\lambda_j+1}^{T_o} \delta_{jt} \qquad$ for $j = 1, 2, \ldots, J$

The remaining constraints are necessary to ensure a proper definition of y_{it} (the number of payloads of type i that are retrieved during year t) and w_{it} (the number of retrieved payloads of type i that are sent up again in year t). We make the following assumptions:

(a) A Tug taking to orbit a payload of type i can only retrieve a payload of the same type, or none at all.

(b) All payloads must remain in orbit at least a given number of years before they are eligible for retrieval.

(c) If a payload of type i is to be retrieved, it can only be retrieved if it is to be sent back up again.

(d) A Tug can retrieve only one payload at a time.

(e) A Tug can retrieve a satellite even if it does not take another into orbit.

(f) A payload must be refurbished for a given number of years before being sent up again.

Let us now define:

$$b_{ij} = \begin{cases} 1 & \text{If Tug } j, \text{ in performance of mission } i, \text{ is also} \\ & \text{capable of retrieving a payload of type } i \\ 0 & \text{otherwise} \end{cases}$$

The first constraint to be defined dictates that the number
of payloads of type i retrieved during year t must not exceed
the total number of payloads of type i sent up during that year by
Tugs capable of retrieval, plus the number of Tug flights sent up in
the "retrieve only" mode. This is given by:

$$(6) \quad y_{it} \leq \sum_{j=1}^{J} [a_{ij} b_{ij} x_{ijt} + \alpha_{ij} z_{ijt}] \quad \text{for } i = 1, 2, \ldots, I$$
$$t = \Lambda + \omega_i + 1, \ldots, T - \sigma_i$$

Note that $y_{it} = 0$ for all i and for $t < \Lambda + \omega_i + 1$ or for $t > T - \sigma_i$.
This will be explained in the next two paragraphs.

The next constraint can be referred to as an "existence"
constraint. It ensures that the total number of payloads of type i
retrieved before or during a particular year cannot exceed the
total number sent up during all prior years. This can be represented
by:

$$(7) \quad \sum_{r=\Lambda+\omega_i+1}^{t} y_{ir} \leq \sum_{j=1}^{J} \sum_{r=\lambda_j+1}^{t-\omega_i} a_{ij} x_{ijr} \quad \text{for } i = 1, 2, \ldots, I$$
$$t = \Lambda + \omega_i + 1, \ldots, T - \sigma_i$$

Note that the summation over time on the left hand side of the in-
equality starts at $r = \Lambda + \omega_i + 1$. The reason for this is that $\Lambda + 1$
represents the first year that any of the Tugs is available for use.
Hence it is not before the year $\Lambda + \omega_i + 1$ that any payload of type i
could be retrieved, and therefore t cannot be less than this value.
Note also that the second summation on the right hand side starts,

for each j, in the year $\lambda_j + 1$. Since the j-th Tug is first available for use in the year $\lambda_j + 1$, this is the first year that Tug j could have sent up a payload of type i.

The next constraint ensures that assumption (c) is imposed. It says that a payload of type i will be retrieved in year t only if it will be sent up again in year $t + \sigma_i$ or later. It is represented by:

$$(8) \qquad y_{it} \le \sum_{j=1}^{J} a_{ij} \sum_{r=t+\sigma_i}^{T} x_{ijr} \qquad\qquad \text{for } i = 1, 2, \ldots, I$$
$$t = \Lambda + \omega_i + 1, \ldots, T - \sigma_i$$

Since the payload of type i will be retrieved only if it will be used σ_i or more years later, the year $T - \sigma_i$ is the last year during which payloads of type i could be retrieved. Hence the variable y_{it} is not defined (or is identically equal to zero) for $t > T - \sigma_i$.

The next set of constraints ensures that the number of retrieved payloads of type i sent up in year t does not exceed the number of such payloads sent up that year. These constraints are represented by

$$(9) \qquad w_{it} \le \sum_{j=1}^{J} a_{ij} x_{ijt} \qquad\qquad \text{for } i = 1, 2, \ldots, I$$
$$t = \Lambda + \omega_i + \sigma_i + 1, \ldots, T$$

Note that w_{it} is not defined for $t < \Lambda + \omega_i + \sigma_i + 1$. Since $\Lambda + 1$ is the first year that a payload of type i could have been sent up, it must

remain in orbit at least ω_i years, and since it takes σ_i years to refurbish the payload, $\Lambda + \omega_i + \sigma_i + 1$ is the first year that a retrieved payload of type i could have been sent up again.

The last set of constraints ensures that the number of retrieved payloads of type i that are sent back up in year t does not exceed the number of retrieved payloads of type i that are available to be sent up again. These are represented by

$$(10) \quad \sum_{r=\Lambda + \omega_i + \sigma_i + 1}^{t} w_{ir} \leq \sum_{r=\Lambda + \omega_i + 1}^{t - \sigma_i} y_{ir} \quad \text{for } i = 1, 2, \ldots, I$$
$$t = \Lambda + \omega_i + 1, \ldots, T$$

Note that assumption (f) is insured by the upper limit of the summation on the right hand side of the inequality. It says that only payloads retrieved at least σ_i years ago can be considered for reuse.

All that remains is to specify which variables are integer constrained. Clearly, it should be the case that all variables are so constrained. But to do so would yield an integer programming problem that would take an inordinately long time to solve. A compromise is warranted, and is suggested to be that all variables except w_{it}, x_{ijt}, y_{it} and z_{ijt} are integer. Specifically, the constraint would be

$$\gamma_{jt} = \text{Non-negative integer} \quad \text{for } j = 1, 2, \ldots J$$
$$t = \lambda_j + 1, \lambda_j + 2, \ldots, T_o$$

$$\delta_{jt} = 0 \text{ or } 1 \qquad \text{for } j = 1, 2, \ldots, J$$

$$t = \lambda_j + 1, \lambda_j + 2, \ldots, T_o$$

$$\epsilon_j = \text{Non-negative integer} \qquad \text{for } j = 1, 2, \ldots, J$$

$$w_{it} \geq 0 \qquad \text{for } i = 1, 2, \ldots, I$$

$$t = \Lambda + \omega_i + \sigma_i + 1, \ldots, T$$

$$x_{ijt} \geq 0 \qquad \text{for } i = 1, 2, \ldots, I$$

$$j = 1, 2, \ldots, J$$

$$t = \lambda_j + 1, \lambda_j + 2, \ldots, T$$

$$y_{it} \geq 0 \qquad \text{for } i = 1, 2, \ldots, I$$

$$t = \Lambda + \omega_i + 1, \ldots, T - \sigma_i$$

$$z_{ijt} \geq 0 \qquad \text{for } i = 1, 2, \ldots, I$$

$$j = 1, 2, \ldots, J$$

$$t = \Lambda + \omega_i + 1, \ldots, T - \sigma_i$$

There is no rigorous justification for the continuity of w, x, y, and z. One cannot speak of a fractional number space flights. However, a rather well-thought-out rationalization can be put forth, as follows. The purpose of this study was to recommend which set of Space Tugs are most cost effective to develop and use, given a particular mission profile. The task was not necessarily to schedule which missions will be performed by which Tugs. Nevertheless, the output from the model would indicate a schedule that

minimizes the total program cost. Since the schedule is not the

primary output of the model, but comes out as a byproduct of the

analysis, the possible non-integer values of w, x, y, and z should

not detract from the main conclusion of the model that a certain set of

Tugs should be developed. While we cannot say that the imposition of

strict integer values for w, x, y, and z would not change the main

conclusion of the most cost-effective set of Tugs, we feel that the

answer arrived at cannot be far off. In any case, it would not

be a difficult task to "integerize" the values of w, x, y, and z

and recompute the total program cost. Thus, we would have a

measure of the maximum error possible in our solution. One

point should be made clear. In the initial years of the mission

profile, the solution would be integer. To see this, let us suppose

that for one particular mission and early year, the number of

flights required is 2 and there are three Tugs capable of carrying

out the mission — concepts numbered 1, 2, and 3. The constraint (1)

would be:

$$X_1 + X_2 + X_3 = 2$$

Not requiring integer values would allow for an infinite number of

solutions to the above equation. However, if $X_1 > 0$, then the

RDT&E costs must be incurred for Tug 1. Similarly, if X_2 or

$X_3 > 0$, so must the RDT&E costs for these be incurred. Thus,

it is most likely, that only one of the three would be non-zero.

In later years, however, when all Tugs that are going to be developed,

have incurred the RDT&E expenses, there would be no way to

avoid a solution such as:

$$X_1 = 0.2, \; X_2 = 0.6, \; X_3 = 1.2$$

It can be argued (as part of the rigorous rationalization) that such a solution should simply be interpreted as meaning that Tug 1 should fly 10 percent of such missions, Tug 2, 30 percent and Tug 3, 60 percent.

At such time as integer programming codes become more efficient and/or computer capacity more available, the integer requirement on w, x, y, and z can be imposed. Until that time, however, there is no choice but to relax that requirement.

RESULTS OF THE ANALYSIS

It was the intention of the Mathematica team to analyze six Tugs using the mixed integer programming model discussed previously. These six were as follows:

1. Large Tank Agena OIS

2. D-1T Centaur OIS

3. LO_2/LH_2 reusable Tug, $W_p = 50K$

4. LO_2/LH_2 reusable Tug, $W_p = 36K$

5. LH_2/LF_2 reusable Tug, $W_p = 48K$

6. $FLOX/CH_4$ reusable Tugs $W_p = 52K$

It was planned first to make a comparison between the two LO_2/LH_2 Tugs, to follow this with an analysis of the same two Tugs plus the Large Tank Agena, and end with an analysis of all six.

However, Mathematica was not able to accomplish this goal fully as it became evident that the cost to run these analyses was too high. The six Tug problems resulted in a mathematical programming problem having approximately 1,400 rows and more than 4,000 variables, ninety of which were integer. By mathematical programming standards, even without the ninety integer variables, this is a large problem. With the inclusion of the integer variables, the problem is massive.

The one problem that was run to completion was the comparison between the two LO_2/LH_2 Tugs. This run took seventy-eight minutes on an IBM 360/65 and resulted in selecting the 50,000 lb version as the only Tug to be developed. This conclusion agreed with other Lockheed/Mathematica study results. One additional contribution of the first run was to come up with a better retrieval schedule than had previously been available. In the earlier version of the retrieval schedule, some retrievals could have been postponed and some could have been avoided altogether. The mixed integer programming problem output corrected these errors.

When an attempt was made to add the Large Tank Agena into the comparison, it soon became evident that the cost to complete this run was in excess of the computer budget. Hence, Mathematica was forced to abandon operational use of the program after demonstrating its general feasibility.

Were the problem to be reduced in size, the running time, of course, would also be reduced. One way to reduce the problem would be to reduce the mission model by combining several missions into one. Another sub-optimal, but nonetheless useful way to use the model would be to dictate the IOC date of each candidate Tug. This would greatly reduce the number of integer variables. If one is interested in analyzing the effects of postponing the IOC

date of the reuseable Tug, a very worthwhile application is to introduce the expendable version in the first year that any mission is to be flown (1979 in our case) and constrain the IOC date of one or more reuseable Tugs to be at least as late as or precisely equal to the date of interest. Such a run would be much less expensive, and would contribute greatly to a total analysis of the Tug. For one thing, it would determine which Tug was most cost effective to develop at the later date (which could be different from that with an early IOC date), and this would then indicate whether or not (and if so, to what extent) it was desirable to maintain the expendable version.

In conclusion, Mathematica found that the mixed integer programming formulation of the Tug problem is too expensive to be efficient in a Tug analysis when used on the full mission model, as was attempted. Nonetheless, when in a slightly sub-optimal way, it does, indeed, offer cost-effective contributions to a complete economic analysis of the Space Tug problem.

OPCHOICE PROGRAM DESCRIPTION

A summary of the mathematical model used for the OPCHOICE computer program follows:

Find values of γ_{jt}, δ_{jt}, w_{it}, x_{ijt}, y_{it}, and z_{ijt} that satisfies the condition:

$$\text{Min} \sum_{j=1}^{J} \sum_{t=\lambda_j + 1}^{T_o} c_j \left[\sum_{s-1}^{\bar{\lambda}_j} P_j (s) v^{t - \lambda_j + s - 2} \right] \delta_{jt}$$

$$+ \sum_{j=1}^{J} \sum_{t=\lambda_j + 1}^{T_o} f_j \left[\sum_{s=1}^{\nu_j + \eta_j} G_j (s) v^{t - \nu_j + s - 2} \right] \nu_{jt}$$

$$+ \sum_{i=1}^{I} \sum_{j=1}^{J} a_{ij} r_{ij} \sum_{t=\lambda_j + 1}^{T} v^{t - 1} x_{ijt} + \sum_{i=1}^{I} \sum_{j=1}^{J} \alpha_{ij} \rho_{ij} \sum_{t=\lambda_j + \omega_i + 1}^{T - \sigma_i} v^{t - 1} z_{ijt}$$

$$- \sum_{i=1}^{I} g_i \sum_{t = \Lambda + \omega_i + \sigma_i + 1}^{T} v^{t-1} w_{it}$$

Subject to

(1) $\quad \displaystyle\sum_{j=1}^{J} a_{ij} x_{ijt} = D_{it} \quad$ for $\ i=1, 2, \ldots, I$

$$t= \Lambda + 1, \ldots, T$$

(2) $\quad \displaystyle\sum_{i=1}^{I} \left[a_{ij} x_{ijt} + \alpha_{ij} z_{ijt} \right] \leq M \sum_{r=\lambda_j + 1}^{t_o} \delta_{jr}$

where $\ t_o = \min (t, T_o)$

for $\ j = 1, 2, \ldots, J \ $ and $\ t = \lambda_j + 1, \ \lambda_j + 2, \ldots, T$

(3) $\quad \displaystyle\sum_{i=1}^{I} \sum_{t=\lambda_j + 1}^{T} \left[a_{ij} x_{ijt} + \alpha_{ij} z_{ijt} \right] \leq N_j e_j \quad$ for $\ j = 1, 2, \ldots, J$

(4) $\quad e_j - \nu_{jt} \leq M(1 - \sigma_{jt})$
$$\text{for } j = 1, 2, \ldots, J$$
$$t = \lambda_j + 1, \lambda_j + 2, \ldots, T_o$$

(5) $\quad e_j \leq M \sum\limits_{t=\lambda_j + 1}^{T_o} \sigma_{jt}$
$$\text{for } j = 1, 2, \ldots, J$$

(6) $\quad y_{it} \leq \sum\limits_{j=1}^{J} \left[a_{ij} b_{ij} x_{ijt} + \alpha_{ij} z_{ijt} \right]$
$$\text{for } i = 1, 2, \ldots, I$$
$$t = \Lambda + \omega_i + 1, \ldots \ T - \sigma_i$$

(7) $\quad \sum\limits_{r = \Lambda + \omega_i + 1}^{t} y_{ir} \leq \sum\limits_{j=1}^{J} \sum\limits_{r=\lambda_j + 1}^{t - \omega_i} a_{ij} x_{ijr}$
$$\text{for } i = 1\ 1, 2, \ldots, I$$
$$t = \Lambda + \omega_i + 1, \ldots, T - \sigma_i$$

(8) $\quad y_{it} \leq \sum\limits_{j=1}^{J} a_{ij} \sum\limits_{r = t + \sigma_i}^{T} x_{ijr}$
$$\text{for } i - 1, 2, \ldots, I$$
$$t = \Lambda + \omega_i + 1, \ldots, T - \sigma_i$$

(9) $\quad w_{it} < \sum\limits_{j=1}^{J} a_{ij} x_{ijt}$
$$\text{for } i = 1, 2, \ldots, I$$
$$t = \Lambda + \omega_i + \sigma_i + 1, \ldots, T$$

(10) $\displaystyle\sum_{r - \Lambda + \omega_i + \sigma_i + 1}^{t} w_{ir} \leq \sum_{r = \Lambda + \omega_i + 1}^{t - \sigma_i} y_{ir}$

$$\text{for } i = 1, 2, \ldots, I$$
$$t = \Lambda + \omega_i + \sigma_i + 1, \ldots, T$$

$\nu_{jt} = \text{non-negative integer}$

$$\text{for } j = 1, 2, \ldots, J$$
$$t = \lambda_j + 1, \lambda_j + 2, \ldots, T_o$$

$\delta_{jt} = 0 \text{ or } 1$

$$\text{for } j = 1, 2, \ldots, J$$
$$t = \lambda_j + 1, \lambda_j + 2, \ldots, T_o$$

$e_j = \text{non-negative integer}$

$$\text{for } j = 1, 2, \ldots, J$$

$w_{it} \leq o$

$$\text{for } i = 1, 2, \ldots, I$$
$$t = \Lambda + \omega_i + \sigma_i + 1, \ldots, T$$

$x_{ijt} \leq 0$

$$\text{for } i = 1, 2, \ldots, I$$
$$j = 1, 2, \ldots, J$$
$$t = \lambda_j + 1, \lambda_j + 2, \ldots, T$$

$y_{it} \leq 0$

$$\text{for } i = 1, 2, \ldots, I$$
$$t = \Lambda + \omega_i + 1, \ldots, T - \sigma_i$$

$z_{ijt} \leq 0$

$$\text{for } i = 1, 2, \ldots, I$$
$$j = 1, 2, \ldots, J$$
$$t = \Lambda + \omega_i + 1, \ldots, T - \sigma_i$$

DICTIONARY OF TERMS

1] a_{ij} = $\begin{cases} 1 & \text{if Tug } j \text{ can perform mission } i, \\ & i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \\ 0 & \text{otherwise} \end{cases}$

2] b_{ij} = $\begin{cases} 1 & \text{if Tug } j, \text{ in performance of mission } i, \text{ is also} \\ & \text{capable of retrieving a payload of type } i, \\ & i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \\ 0 & \text{otherwise} \end{cases}$

3] c_j = Undiscounted RDT & E costs for Tug of type j, for $j = 1, 2, \ldots, J$

4] D_{it} = Number of satellites of type i which must be put into orbit during year t. $i = 1, 2, \ldots, I$; $t = \Lambda + 1, \Lambda + 2, \ldots, T$

5] f_j = Undiscounted investment cost to make one Tug of type j, $j = 1, 2, \ldots, J$

6] g_i = Undiscounted benefit realized by reusing a payload of type i rather than making it again, $i = 1, 2, \ldots, I$

7] I = The number of different missions in the mission profile

8] J = The number of different Tug configurations under consideration

9] M = Any large number (for integer programming purposes)

10] N_j = The number of flights that can be made by a Tug of type j (also called the mean life) $j = 1, 2, \ldots, J$

11] $P_j(s)$ = Spreading function for RDT & E costs for Tug j,
$j = 1, 2, \ldots, J$

12] $Q_j(s)$ = Spreading function for investment costs for Tug j,
$j = 1, 2, \ldots, J$

13] r_{ij} = Undiscounted recurring cost for Tug j to perform
mission i, $i = 1, 2, \ldots, I; j = 1, 2, \ldots, J$

14] T = Total number of years in the mission profile

15] T_o = Total number of years during which Tugs could
be developed (i.e., after a given date, we
assume that no more Tugs can be made)

16] v = $(1 + d)^{-1}$, where d = social discount rate

17] w_{it} = The number of retrieved payloads of type i that
are sent up again in year t, $i = 1, 2, \ldots, I$;
$t = \Lambda + \omega_i + \sigma_i + 1, \ldots, T$

18] x_{ijt} = The number of flights in year t flown by Tug j
in order to perform mission i, $i = 1, 2, \ldots, I$;
$j = 1, 2, \ldots, J; t = \lambda_j + 1, \lambda_j + 2, \ldots, T$

19] y_{it} = The number of payloads of type i recovered in
year t, $i = 1, 2, \ldots, I; t = \Lambda + \omega_i + 1, \ldots, T - \sigma_i$

20] z_{ijt} = The number of flights in year t that Tug j will
perform to retrieve payload i when Tug is flown
in the "retrieve only" mode $i = 1, 2, \ldots, I$;
$j = 1, 2, \ldots, J; t = \lambda_j + \omega_i + 1, \ldots, T - \sigma_i$

21] α_{ij} = $\begin{cases} 1 \text{ if Tug j can retrieve payload i when Tug j is} \\ \quad \text{flown in the "retrieve only" mode } i = 1, 2, \ldots, I; \\ \quad j = 1, 2, \ldots, J \\ 0 \text{ otherwise} \end{cases}$

22] ν_{jt} = $\begin{cases} \epsilon_j & \text{if the Tugs are available for the first time in} \\ & \text{year } t, \ j = 1, 2, \ldots, J; \ t = \lambda_j + 1, \ \lambda_j + 2, \ldots, T_0 \\ 0 & \text{otherwise} \end{cases}$

23] δ_{jt} = $\begin{cases} 1 & \text{if Tug } j \text{ is to be developed and available for} \\ & \text{the first time in year } t, \ j = 1, 2, \ldots, J; \\ & t = \lambda_j + 1, \ \lambda_j + 2, \ldots, T_0 \\ 0 & \text{otherwise} \end{cases}$

24] ϵ_j = Number of Tugs of type j which are produced

25] η_j = Number of years beyond date of availability that investment costs for Tug j will be spread $j = 1, 2, \ldots, J$

26] λ_j = Number of years required for RDT & E for Tug j, $j = 1, 2, \ldots, J$

27] Λ = $\min_j \lambda_j$

28] ν_j = Number of years required to build a Tug of type j, $j = 1, 2, \ldots, J$

29] ρ_{ij} = Undiscounted recurring costs for Tug j to retrieve payload i when Tug is flown in the "retrieve only" mode, $i = 1, 2, \ldots, I; \ j = 1, 2, \ldots, J$

30] σ_i = The number of years after a payload is recovered that it can be reused, $i = 1, 2, \ldots, I$

31] ω_i = The number of years that a payload to type i must remain in orbit